ECE171A: Linear Control System Theory
Lecture 1: Introduction

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JACOBS SCHOOL OF ENGINEERING
Electrical and Computer Engineering

Outline

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Overview

- **ECE 171A: Linear Control System Theory** focuses on modeling, analysis, and design of single-input single-output linear time-invariant control systems with emphasis on frequency-domain techniques
- Topics:
  - **Modeling**: ordinary differential equations, linear time-invariant systems, first-order and second-order systems, block diagrams and signal flow graphs
  - **Analysis**: transient and steady-state behavior, Laplace transform, stability, root locus, frequency response, Bode plots, Nyquist plots, Nichols plots
  - **Design**: PID control, loop shaping

Prerequisites

- Prerequisites
  - **Introductory physics**: Newton's law, Ohm's law, Kirchhoff's voltage and current laws
  - **Calculus**: derivatives, integration, exponential function, Taylor series
  - **Programming experience** with MATLAB, Python, or similar language
  - Optional but helpful: ordinary differential equations, linear algebra
- Courses that fulfill these prerequisites: **ECE 45** or **MAE 40**

Teaching Team

- Instructor:
  - Nikolay Atanasov
  - Assistant Professor, ECE department
  - Email: natanasov@ucsd.edu

- Teaching Assistant:
  - Shrey Kansal
  - Graduate student, MAE department
  - Email: skansal@ucsd.edu

Discussion Session and Office Hours

- **Discussion Session**
  - Mondays, 3:00 pm - 4:00 pm, in CENTR 222.
  - Optional material and examples supplementing the lectures will be covered.
  - You are encouraged to ask questions and start a discussion!
- **Office Hours**
  - Fridays, 11:00 am - 12:00 pm on Zoom (see Canvas for information).
  - No new material will be covered.
  - The focus will be on answering your questions and helping you with the material.
- Additional office hours by appointment

It would be great if most, if not all, of you attend the discussion and office hours together. Even if you do not have questions, you can help answer other students' questions and create a supportive community for this class!

Assignments and Grading

- Course website: `https://natanaso.github.io/ece171a`
- Includes links to:
  - **Canvas**: course password, lecture recordings
  - **Gradescope**: homework submission and grades
  - **Piazza**: discussion and class announcements (please check regularly!)
- Assignments:
  - Academic integrity quiz on Canvas (due by Oct 05)
  - 7 homework sets (45% of grade)
  - Midterm exam (25% of grade): calculator + single-sided cheat sheet
  - Final exam (30% of grade): calculator + double-sided cheat sheet
- Grading:
  - A standard grade scale (e.g., 93%+ = A) will be used with a curve based on the class performance (e.g., if the top students have grades in the 83%-86% range, then this will correspond to letter grade A)
  - **No late policy**: homework submitted past the deadline will not be accepted due to our tight course schedule

References

- **Primary Textbook**:

  Karl J. Åström and Richard M. Murray, *Feedback Systems: An Introduction for Scientists and Engineers*, 2008

  Available at: `https://fbswiki.org/`

- **Additional References**: listed on the course website (not required)

Course Schedule (Tentative)

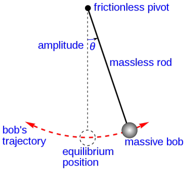
Week	Date	Lecture	Material	Assignment
1	Sep 26	Introduction	Ch. 1	
	Sep 28	Feedback Control Systems	Ch. 2	
2	Oct 03	Discussion		
	Oct 03	System Modeling	Ch. 3	
	Oct 05	Solving ODEs	Ch. 5	HW1
3	Oct 10	Discussion		
	Oct 10	Catch-up		
	Oct 12	Laplace Transform, Transfer Function	Ch. 9	HW2
4	Oct 17	Discussion		
	Oct 17	Block Diagram, Signal Flow Graph		
	Oct 19	Stability, Routh-Hurwitz	Ch. 6	HW3
5	Oct 24	Discussion		
	Oct 24	Transient and Steady-state Response	Ch. 6	
	Oct 26	Root Locus	Ch. 10	
6	Oct 31	Discussion		
	Oct 31	Midterm Exam		

Check the course website for reading material and schedule updates:
`https://natanaso.github.io/ece171a`

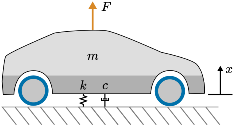
Outline

What is a dynamical system?

- A **dynamical system** is a system whose behavior changes over **time**, often in response to internal or external stimulation

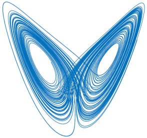


(a) Pendulum

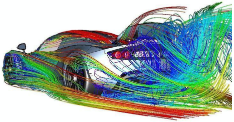


(b) Car suspension

(c) Water flow



(d) Atmospheric convection (Lorenz system)

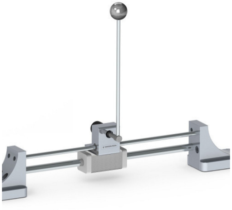


(e) Fluid dynamics

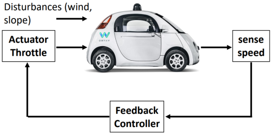
(f) Sync of fish

What is a control system?

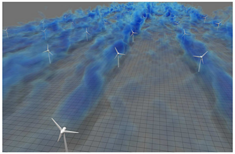
- A **control system** is an interconnection of components (dynamical systems) that provides a desired response
- A **controller** is a component of a control system that modifies the overall system behavior



(a) Inverted pendulum



(b) Cruise control



(c) Wind farm

Control System

- Modern control systems include physical and cyber components

- A **physical component** is a mechanical, electrical, fluid, or thermal device acting as a sensor, actuator, or embedded system component:
  - A **sensor** is a device that provides measurements of a signal of interest (*output*)
  - An **actuator** is a device that alters the configuration of the system or its environment (*input*)

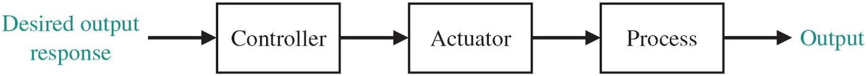
- A **cyber component** is a software node that executes a specific function

- **Control system engineering** focuses on:
  - modeling cyber-physical systems,
  - analyzing system behavior,
  - designing control techniques to achieve desired behavior

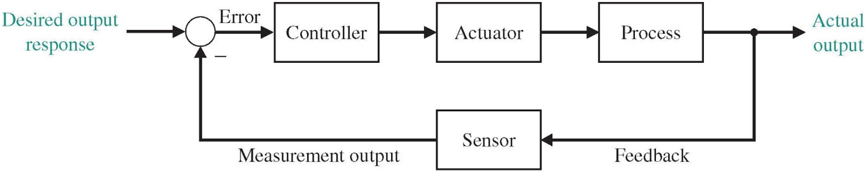
- **Performance characteristics**: stability, transient and steady-state tracking, rejection of external disturbances, robustness to modeling uncertainties, etc.

Open-loop vs Closed-loop Control Systems

- An **open-loop (feedforward) control system** utilizes a controller without measurement feedback of the system output

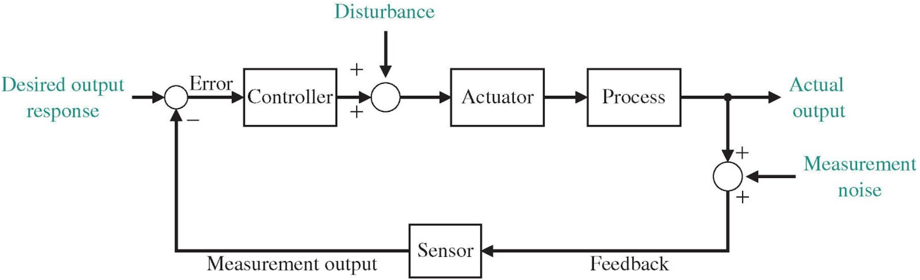


- A **closed-loop (feedback) control system** utilizes a controller with measurement feedback of the system output



Noise and Modeling Errors

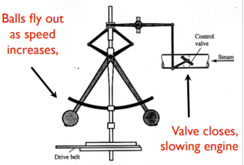
- A closed-loop control system uses the measurement feedback to compute and reduce the **error** between the desired and measured output
- A closed-loop control systems may attenuate the effects of **process noise** (disturbance), **measurement noise**, and **modeling errors**

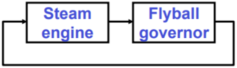


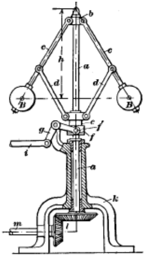
Feedback Control Examples

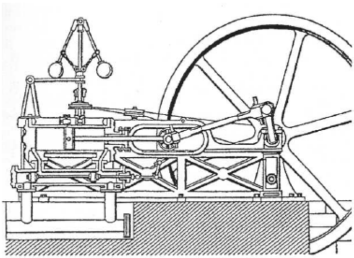
Flyball Governor (1788)

- regulate speed of a steam engine
- reduces the effect of load variations
- major advance of industrial revolution









Feedback Control Examples

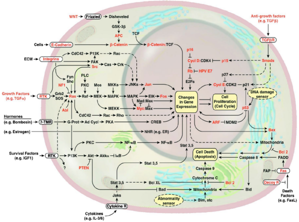
(a) Drones

(b) Autonomous vehicles

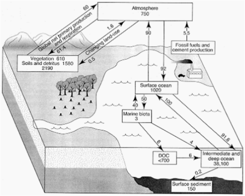
(c) Rockets

(d) Aircraft

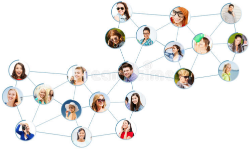
Feedback Control Examples



(e) Biological systems



(f) Environmental systems

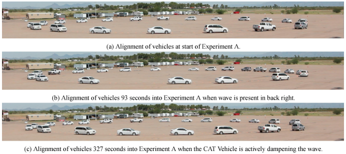


(g) Social networks

(h) Finance market

Feedback Control Examples

(a) Robot parkour



(b) Traffic wave control

(c) Autonomous flight

Outline

Control System Modeling

- A mathematical model is a key element in the design and analysis of control systems
- Dynamic behavior is described by **ordinary differential equations** (ODEs)

$$\frac{d}{dt}y(t) + a(t)y(t) = u(t)$$

- The relationship between the variables and their derivatives in an ODE may be **linear** or **nonlinear**
- Nonlinear ODEs are often approximated using **linearization** because linear ODEs are much easier to analyze
- The coefficients of an ODE may be **time-invariant** or **time-varying**
- This class will focus on **linear time-invariant** (LTI) ODE systems

Differential Equations of Physical Systems

- Physical systems from different domains (electrical, mechanical, fluid, thermal) contain elements that share similar roles

Variable	Electrical	Mechanical	Fluid	Thermal
Through	Current	Force, Torque	Flow rate	Flow rate
Across	Voltage	Velocity	Pressure	Temperature
Inductive	Inductance	Inverse Stiffness	Inertia	–
Capacitive	Capacitance	Mass, Moment of Inertia	Capacitance	Capacitance
Resistive	Resistance	Friction	Resistance	Resistance

- The dynamic behavior of these elements is described by physical laws, such as Kirchhoff's laws or Newton's laws, enabling an ODE description of the system

Through and Across Element Variables

Table 2.1 Summary of Through- and Across-Variables for Physical Systems

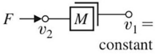
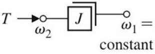
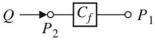
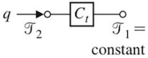
System	Variable Through Element	Integrated Through-Variable	Variable Across Element	Integrated Across-Variable
Electrical	Current, i	Charge, q	Voltage difference, v_{21}	Flux linkage, λ_{21}
Mechanical translational	Force, F	Translational momentum, P	Velocity difference, v_{21}	Displacement difference, y_{21}
Mechanical rotational	Torque, T	Angular momentum, h	Angular velocity difference, ω_{21}	Angular displacement difference, θ_{21}
Fluid	Fluid volumetric rate of flow, Q	Volume, V	Pressure difference, P_{21}	Pressure momentum, γ_{21}
Thermal	Heat flow rate, q	Heat energy, H	Temperature difference, $\mathcal{T}_{21}$	

Inductive Elements

Table 2.2 Summary of Governing Differential Equations for Ideal Elements

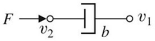
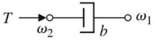
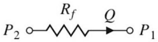
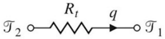
Type of Element	Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Inductive storage	Electrical inductance	$v_{21} = L\frac{di}{dt}$	$E = \frac{1}{2}Li^2$	
	Translational spring	$v_{21} = \frac{1}{k}\frac{dF}{dt}$	$E = \frac{1}{2}\frac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \frac{1}{k}\frac{dT}{dt}$	$E = \frac{1}{2}\frac{T^2}{k}$	
	Fluid inertia	$P_{21} = I\frac{dQ}{dt}$	$E = \frac{1}{2}IQ^2$	

Capacitive Elements

Table 2.2 Summary of Governing Differential Equations for Ideal Elements				
Type of Element	**Physical Element**	**Governing Equation**	**Energy *E* or Power $\mathcal{P}$**	**Symbol**
Capacitive storage	Electrical capacitance	$i = C\dfrac{dv_{21}}{dt}$	$E = \dfrac{1}{2}C{v_{21}}^2$	
	Translational mass	$F = M\dfrac{dv_2}{dt}$	$E = \dfrac{1}{2}M{v_2}^2$	
	Rotational mass	$T = J\dfrac{d\omega_2}{dt}$	$E = \dfrac{1}{2}J{\omega_2}^2$	
	Fluid capacitance	$Q = C_f\dfrac{dP_{21}}{dt}$	$E = \dfrac{1}{2}C_f{P_{21}}^2$	
	Thermal capacitance	$q = C_t\dfrac{d\mathcal{T}_2}{dt}$	$E = C_t\mathcal{T}_2$	

Resistive Elements

Table 2.2 Summary of Governing Differential Equations for Ideal Elements

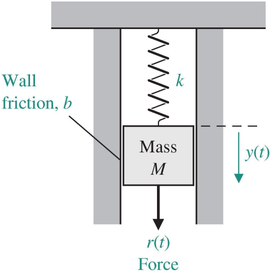
Type of Element		Physical Element	Governing Equation	Energy E or Power $\mathcal{P}$	Symbol
Energy dissipators	{	Electrical resistance	$i = \frac{1}{R} v_{21}$	$\mathcal{P} = \frac{1}{R} {v_{21}}^2$	
		Translational damper	$F = b v_{21}$	$\mathcal{P} = b {v_{21}}^2$	
		Rotational damper	$T = b \omega_{21}$	$\mathcal{P} = b {\omega_{21}}^2$	
		Fluid resistance	$Q = \frac{1}{R_f} P_{21}$	$\mathcal{P} = \frac{1}{R_f} {P_{21}}^2$	
		Thermal resistance	$q = \frac{1}{R_t} \mathcal{T}_{21}$	$\mathcal{P} = \frac{1}{R_t} \mathcal{T}_{21}$	

Spring-Mass-Damper Example

- The behavior of a spring-mass-damper system is described by Newton's second law:

$$M\frac{d^2y(t)}{dt^2} + \underbrace{b\frac{dy(t)}{dt}}_{\text{viscous damper}} + \underbrace{ky(t)}_{\text{spring force}} = \underbrace{r(t)}_{\text{input force}}$$

- The mass displacement $y(t)$ satisfies a second-order linear time-invariant (LTI) ordinary differential equation (ODE)



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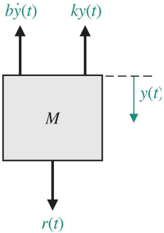

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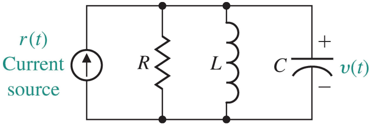
Parallel RLC Circuit Example

- The behavior of an electrical RLC circuit is described by Kirchhoff's current law:

$$r(t) = i_R(t) + i_L(t) + i_C(t)$$



- Parallel devices have the same voltage $v(t)$:
  - Resistor: $v(t) = Ri_R(t)$
  - Inductor: $v(t) = L\frac{di_L(t)}{dt}$
  - Capacitor: $i_C(t) = C\frac{dv(t)}{dt}$

- The inductor current $i_L(t)$ satisfies a second-order LTI ODE:

$$CL\frac{d^2 i_L(t)}{dt^2} + \frac{L}{R}\frac{di_L(t)}{dt} + i_L(t) = r(t)$$

Laplace Transform

- The behavior of an LTI ODE system may be studied in the **time domain** ($t \in \mathbb{R}$) or in the **complex (frequency) domain** ($s = \sigma + j\omega \in \mathbb{C}$)

- The **Laplace transform** converts an **LTI ODE** in the time domain into a **linear algebraic equation** in the complex domain

- Example:
  - Time domain LTI ODE:

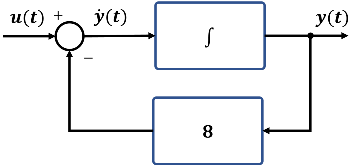
    $$\frac{d}{dt}y(t) + 8y(t) = u(t)$$

  - Laplace domain linear algebraic eq.:

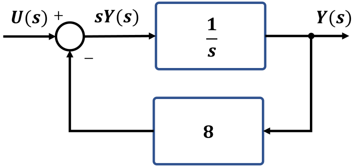
    $$sY(s) - y(0) + 8Y(s) = U(s)$$

Block Diagram

- The system components may be visualized as a **block diagram**
- A **block** represents the input-output relationship of a system component
- To represent multi-component systems, the blocks are interconnected
- Signals may be added or subtracted using **summing points**
- A block diagram may represent a system in the time domain or in the complex domain
- Time domain:

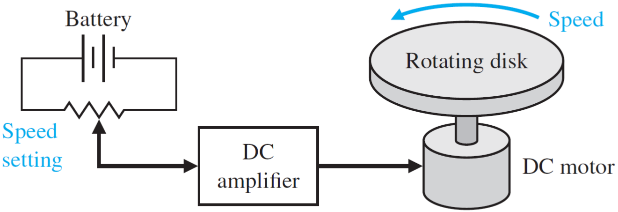


- Laplace domain:



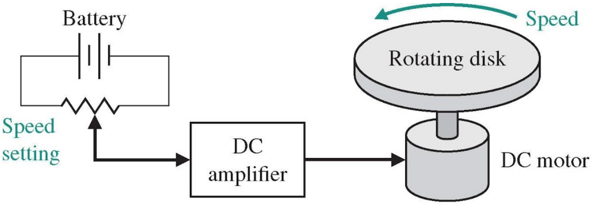
Example: Rotating Disk Speed Control

- Line-cell imaging in biomedical applications use spinning disk conformal microscopes
- Objective: design a controller for a rotating disk system to ensure the speed of rotation is within a specified percentage of a desired speed
- System components:
  - DC motor: provides speed proportional to the applied voltage
  - Battery source: provides voltage proportional to the desired speed
  - DC amplifier: amplifies the battery voltage to meet the motor voltage requirements
  - Tachometer: provides output voltage proportional to the speed of its shaft



(a)

Open-Loop Rotating Disk System



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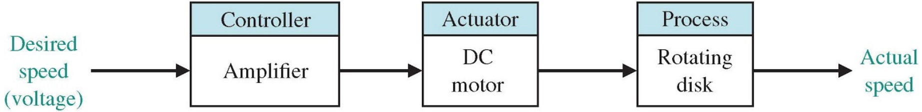

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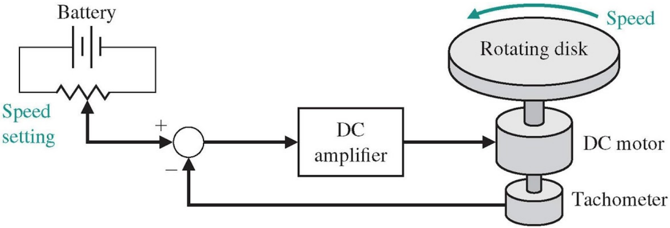
Closed-Loop Rotating Disk System



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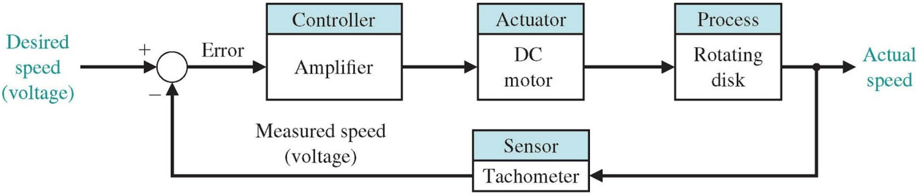


(b)

Control System Analysis

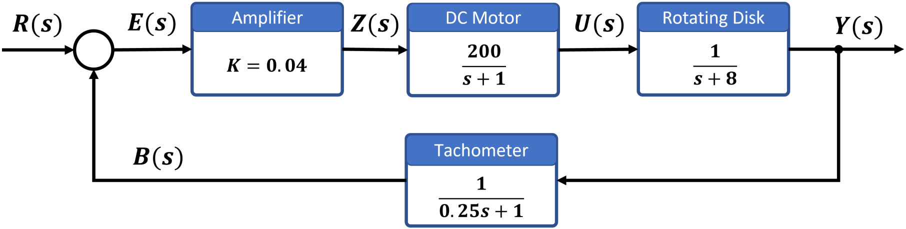
- The system components are described using LTI ODEs

- Time domain:
  - Desired speed: $r(t)$
  - Amplifier: $z(t) = Kr(t)$
  - DC motor: $\dot{u}(t) + u(t) = 200z(t)$
  - Rotating disk: $\dot{y}(t) + 8y(t) = u(t)$
  - Tachometer: $\dot{b}(t) + 4b(t) = 4y(t)$

- Laplace domain:
  - Desired speed: $R(s)$
  - Amplifier: $Z(s) = KR(s)$
  - DC motor: $U(s) = \frac{200}{s+1}Z(s)$
  - Rotating disk: $Y(s) = \frac{1}{s+8}U(s)$
  - Tachometer: $B(s) = \frac{4}{s+4}Y(s)$

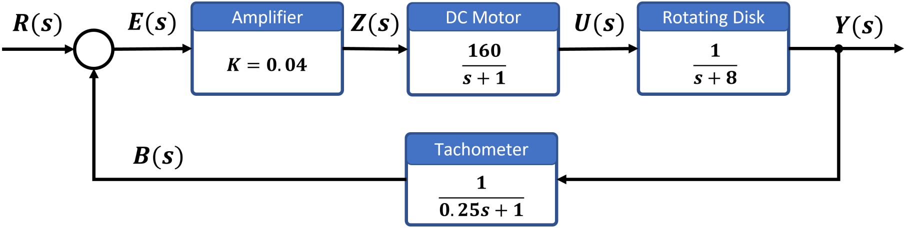
- We will study how to choose the amplifier gain K to ensure that system output $y(t)$ tracks a desired reference signal $r(t)$

Nominal Rotating Disk System



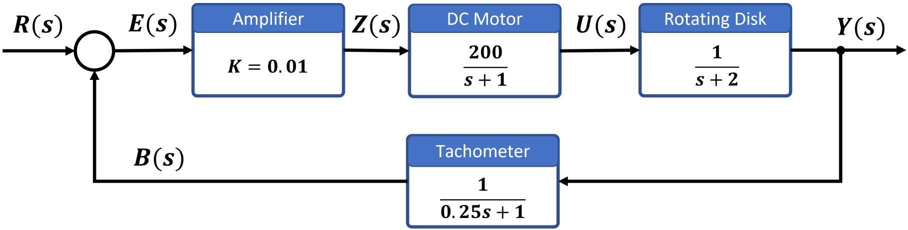
- A nominal model aims to capture the system behavior accurately but parameter errors or disturbances might be present
- Closed-loop control becomes important when there are parameter errors and disturbances

Low-gain Rotating Disk System



- The DC motor gain might be different in the real system (e.g., 160) compared to the nominal model (e.g., 200)

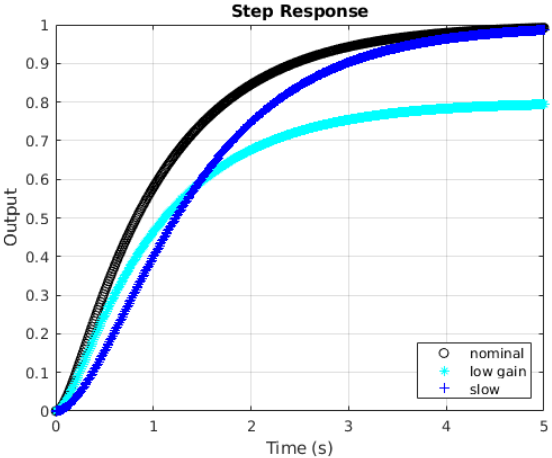
Slow Rotating Disk System



- The disk might rotate slower in the real system (e.g., $\dot{y}(t) + 2y(t) = u(t)$) compared to the nominal model (e.g., $\dot{y}(t) + 8y(t) = u(t)$)

Open-loop Step Response

- Without feedback, the real system response might be different than what was planned



Closed-loop Step Response

- Feedback improves the sensitivity to parameter errors and disturbances
- Despite the advantages, feedback architectures need to be designed carefully to avoid oscillations and steady-state error