ECE171A: Linear Control System Theory
Lecture 2: Feedback Control Principles

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Outline

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Advantages of Feedback Control

- **Disturbance attenuation**:
  - Closed-loop control reduces the effect of disturbances and noise in the system response
- **Robustness to parameter variations**:
  - Closed-loop control reduces the sensitivity of the system response to variations in the model parameters
  - Accurate control may be achieved with imprecise components
- **Dynamic behavior shaping**:
  - Closed-loop control may widen the range in which a system behaves linearly
  - Closed-loop control allows the system output to track a desired reference signal

Disadvantages of Feedback Control

- **Increased system complexity**:
  - Sensing components are necessary for feedback control, which may be expensive and introduces noise
- **Loss of gain**:
  - The forward gain in a closed-loop system is smaller by a certain factor than the forward gain of an open-loop system
  - The gain is decreased by the same factor that reduces the sensitivity to parameter variations and disturbances
  - In practice, the advantage of increased robustness outweighs the loss of control gain
- **Potential for instability**:
  - Closed-loop control may lead to system instability, even if the open-loop system is stable

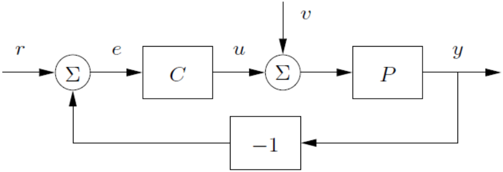
Examples of Feedback Control Use

- Feedback control was used by James Watt to make steam engines run at constant speed in spite of varying load (industrial revolution)
- Feedback control was used by electrical engineers to make water-turbine generators deliver electricity with constant frequency and voltage.
- Feedback control is commonly used to alleviate effects of disturbances in the process industry, for machine tools, and for engine and cruise control in cars.
- The human body exploits feedback to keep body temperature, blood pressure, and other important variables constant.
- **Servo problem**: a major application of feedback control is to make a system's output follow a desired reference signal
  - Examples: car steering, satellite tracking with an antenna, audio amplifiers, industrial robots

Outline

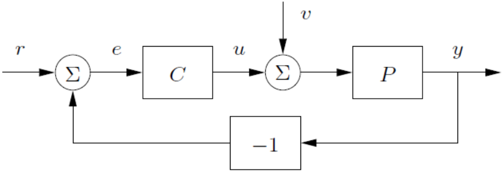
Example: Nonlinear Static System

- Automatic control has had significant impact on industrial automation, e.g., for process control in chemical plants
- The dynamical system to be controlled is often referred to as **plant**



- Reference signal: $r(t)$
- Controller: C
- Plant: P
- Summing point: Σ
- Input: $u(t)$
- Disturbance: $v(t)$
- Output: $y(t)$
- Error: $e(t)$

Example: Nonlinear Static System



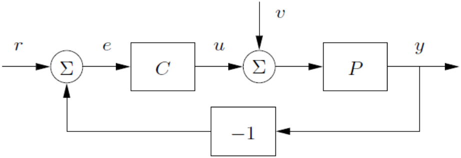
- **Plant** P: consider a static system (no dynamics and no ODE description):

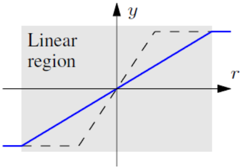
$$y = \mathrm{sat}(x) := \begin{cases} -1 & \text{if } x \leq -1 \\ x & \text{if } |x| < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$

- **Controller** C: consider a controller with constant gain $k > 0$:

$$u = ke$$

Dynamic Behavior Shaping





- Assume no disturbances for now: $v \equiv 0$
- **Open-loop system**: combination of C and P with no feedback:

$$y = \mathrm{sat}(kr) \quad \Rightarrow \quad \text{linear range: } |r| < 1/k$$

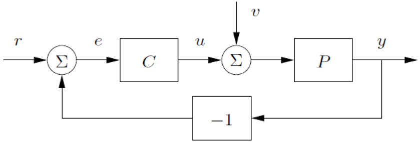
- **Closed-loop system**: combination of C and P with feedback:

$$\left.\begin{aligned} y &= \mathrm{sat}(u) \\ u &= k(r-y) \end{aligned}\right\} \quad \Rightarrow \quad y = \mathrm{sat}(k(r-y))$$

$$\Rightarrow \quad y = \mathrm{sat}\left(\frac{k}{k+1} r\right) \quad \Rightarrow \quad \text{linear range: } |r| < \frac{k+1}{k}$$

Observation 1: Feedback control **widens** the linear range of the system by a factor of $k+1$ compared to the open-loop system

Robustness to Parameter Variations

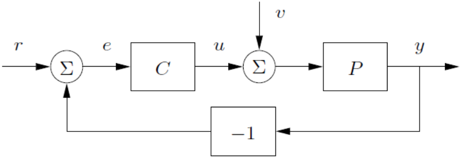


Parameter sensitivity: quantifies the change in system behavior due to change in the system parameters

- **Open-loop system**:
  - In the linear range: $y = kr$
  - It follows that $\frac{dy}{dk} = r = \frac{y}{k} \quad \Rightarrow \quad \frac{dy}{y} = \frac{dk}{k}$
  - **Sensitivity**: 10% change in k leads to 10% change in output
- **Closed-loop system**:
  - In the linear range: $y = \frac{k}{k+1} r$
  - It follows that $\frac{dy}{dk} = \frac{1}{(k+1)^2} r = \frac{1}{(k+1)} \frac{y}{k} \quad \Rightarrow \quad \frac{dy}{y} = \frac{1}{(k+1)} \frac{dk}{k}$
  - **Sensitivity**: for $k = 100$, 10% change in k leads to $\approx 0.1\%$ change in output

Observation 2: Feedback control **reduces the sensitivity** to gain variations by a factor of $k + 1$.

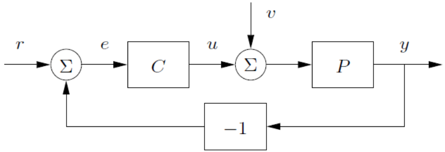
Disturbance Attenuation



- Suppose now that the system is subject to a disturbance signal v
- Assume $r \equiv 0$ for simplicity
- **Open-loop system**:
  - With $r \equiv 0$, $y = \mathrm{sat}(v)$
  - In the linear range, disturbances are passed through with no attenuation
- **Closed-loop system**:
  - With $r \equiv 0$, $y = \mathrm{sat}(v - ky) \quad \Rightarrow \quad y = \mathrm{sat}\left(\frac{v}{k+1}\right)$
  - In the linear range, disturbances are attenuated by a factor of $k + 1$

Observation 3: Feedback control **reduces the effect of disturbances** in the linear range by a factor of $k + 1$.

Summary



- Static plant P:
$$y = \text{sat}(x) := \begin{cases} -1 & \text{if } x \leq -1 \\ x & \text{if } |x| < 1 \\ 1 & \text{if } x \geq 1 \end{cases}$$
- Constant-gain controller C:
$$u = ke, \qquad k > 0$$

Feedback control

- 1) *increases* the range of linearity of the system,
- 2) *decreases* the sensitivity of the system response to parameter variations,
- 3) *attenuates* the effect of disturbances.

The **trade-off** is that

- 1) output sensing is required,
- 2) the closed-loop gain is decreased by a factor of $k+1$:

open-loop:

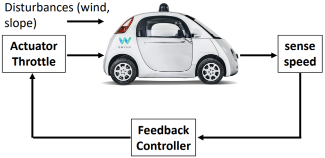
$$y = \text{sat}(kr)$$

closed-loop:

$$y = \text{sat}\left(\frac{k}{k+1} r\right)$$

Outline

Example: Cruise Control System



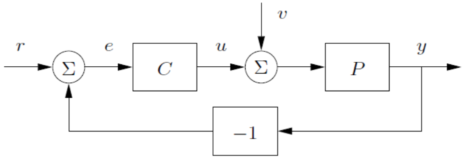
- A cruise controller aims to maintain constant velocity in the presence of disturbances caused by the road slope, friction, air drag, etc.

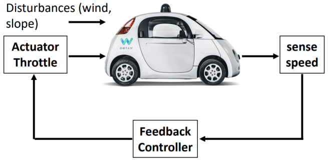
- **Variables**:
  - Desired speed (reference): $r(t)$
  - Actual speed (output): $y(t)$
  - Engine force (input): $u(t)$
  - Mass (parameter): m
  - Disturbances:
    - Road slope: $F_{road} = -mg\sin(\theta)$
    - Air drag: $F_{drag} = -\delta y(t)$

- **System model**:

$$m\dot{y}(t) = u(t) - \delta y(t) - mg\sin(\theta)$$

Example: Cruise Control System





- **Plant** P:

$$m\dot{y}(t) = u(t) - \delta y(t) - mg\sin(\theta)$$

- **Controller** C: design $u(t)$ using reference $r(t)$ and output $y(t)$
- **Performance criteria**:
  - **Stable response**
    - Steady-state velocity approaches desired velocity
    - Smooth response with no overshoot or oscillations
  - **Disturbance rejection**
    - Effect of disturbances (e.g., slope θ) approaches zero over time
  - **Robustness**
    - System response is invariant to variations in the parameters (e.g., mass m)

Closed-loop Control

- **System model**:

$$m\dot{y}(t) = u(t) - \delta y(t) - mg\sin(\theta)$$

- **Closed-loop control**:
  - $u(t)$ designed using the error signal $e(t) = r(t) - y(t)$
  - **P (Proportional)** control:

$$u(t) = k_{\mathrm{p}} e(t)$$

  - **I (Integral)** control:

$$u(t) = k_{\mathrm{i}} \int_0^t e(t)dt$$

  - **D (Derivative)** control:

$$u(t) = k_{\mathrm{d}} \frac{d}{dt} e(t)$$

  - **PID control**:

$$u(t) = k_{\mathrm{p}} e(t) + k_{\mathrm{i}} \int_0^t e(t)dt + k_{\mathrm{d}} \frac{d}{dt} e(t)$$

Open-loop Control

- **System model**:

$$m\dot{y}(t) = u(t) - \delta y(t) - mg\sin(\theta)$$

- **Open-loop control**:
  - $u(t)$ is designed using reference $r(t)$ and initial condition $y(0) = y_0$ but no measurements of the output $y(t)$
  - Approximate the error using y_0 and some function $a(t)$:

$$e(t) \approx r(t) - a(t)y_0$$

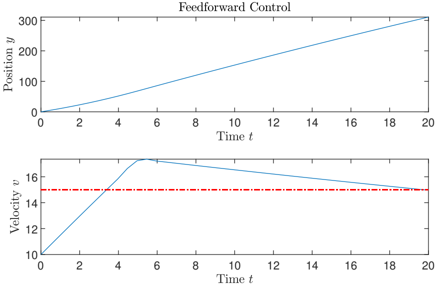
  - Use PID control with the approximate error

Open-loop P Control Simulation

- Parameters: $r(t) \equiv 15$ m/s, $y_0 = 10$ m/s, $m = 500$ kg, $\delta = 0.5$, $\theta = 0°$
- Matlab ODE45 function: `[t,y] = ode45(odefun,tspan,y0)`
- **Case 1**:

$$k_{\mathrm{p}} = 160, \quad a(t) = \begin{cases} 1 & 0 \le t \le 5 \\ \frac{3}{2} & t > 5 \end{cases}$$

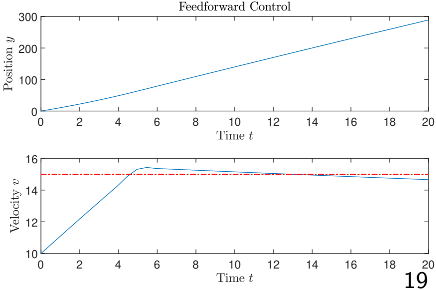
$$u(t) = \begin{cases} 800 & 0 \le t \le 5 \\ 0 & t > 5 \end{cases}$$



- **Case 2**:

$$k_{\mathrm{p}} = 120, \quad a(t) = \begin{cases} 1 & 0 \le t \le 5 \\ \frac{35}{24} & t > 5 \end{cases}$$

$$u(t) = \begin{cases} 600 & 50 \le t \le 5 \\ 0 & t > 5 \end{cases}$$

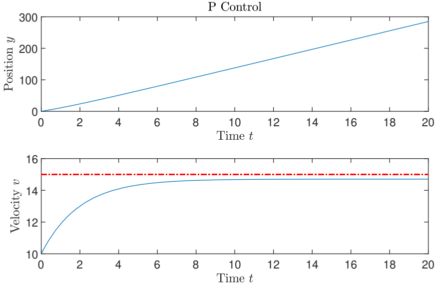


Closed-loop P Control Simulation

- Parameters: $r(t) \equiv 15$ m/s, $y_0 = 10$ m/s, $m = 500$ kg, $\delta = 0.5$

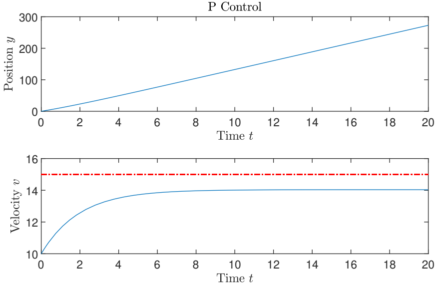
- **Case 1**: flat road $\theta = 0^\circ$

  $k_\mathrm{p} = 250 \quad u(t) = k_\mathrm{p} e(t)$



- **Case 2**: uphill $\theta = 2^\circ$

  $k_\mathrm{p} = 250 \quad u(t) = k_\mathrm{p} e(t)$



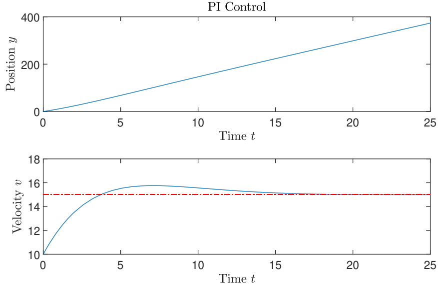
Closed-loop PI Control Simulation

- Parameters: $r(t) \equiv 15$ m/s, $y_0 = 10$ m/s, $m = 500$ kg, $\delta = 0.5$

- **Case 1**: flat road $\theta = 0°$

  $k_{\rm p} = 250, \quad k_{\rm i} = 50$

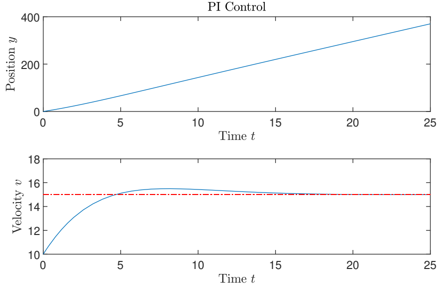
  $$u(t) = k_{\rm p} e(t) + k_{\rm i} \int_0^t e(t) dt$$



- **Case 2**: uphill $\theta = 2°$

  $k_{\rm p} = 250, \quad k_{\rm i} = 50$

  $$u(t) = k_{\rm p} e(t) + k_{\rm i} \int_0^t e(t) dt$$



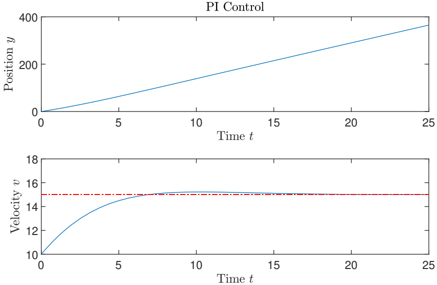
Disturbance Attenuation with PI Control

- Parameters: $r(t) \equiv 15$ m/s, $y_0 = 10$ m/s, $m = 500$ kg, $\delta = 0.5$

- **Case 1**: uphill $\theta = 5^\circ$

  $k_\mathrm{p} = 250, \quad k_\mathrm{i} = 50$

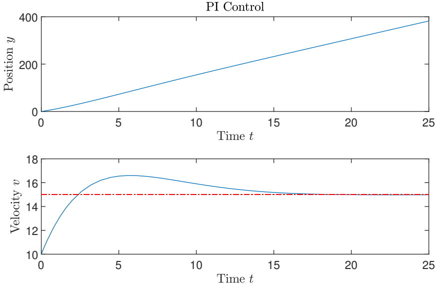
  $$u(t) = k_\mathrm{p} e(t) + k_\mathrm{i} \int_0^t e(t)dt$$



- **Case 2**: downhill $\theta = -5^\circ$

  $k_\mathrm{p} = 250, \quad k_\mathrm{i} = 50$

  $$u(t) = k_\mathrm{p} e(t) + k_\mathrm{i} \int_0^t e(t)dt$$



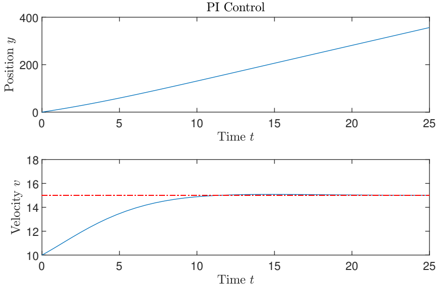
Disturbance Attenuation with PI Control

- Parameters: $r(t) \equiv 15$ m/s, $y_0 = 10$ m/s, $m = 500$ kg, $\delta = 0.5$

- **Case 3**: uphill $\theta = 10°$

  $k_{\mathrm{p}} = 250, \quad k_{\mathrm{i}} = 50$

  $$u(t) = k_{\mathrm{p}} e(t) + k_{\mathrm{i}} \int_0^t e(t) dt$$

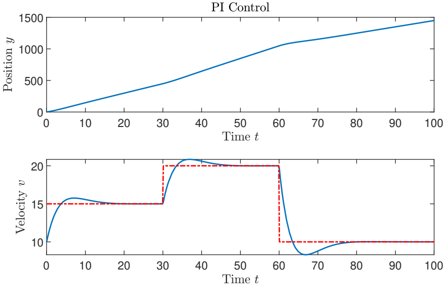


Disturbance attenuation: The same PI controller achieves *zero steady-state error*, i.e., $e(t) \to 0$, despite the presence of an *unknown* disturbance θ.

Dynamic Behavior Shaping with PI Control

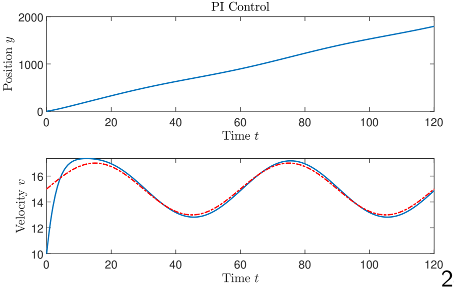
- Parameters: $y_0 = 10$ m/s, $m = 500$ kg, $\delta = 0.5$, $\theta = 0^\circ$
- Closed-loop PI control with $k_\mathrm{p} = 250$ and $k_\mathrm{i} = 50$
- **Case 1**: piecewise-constant reference

$$r(t) = \begin{cases} 15 m/s & t \leq 30 \\ 20 m/s & 30 < t \leq 60 \\ 10 m/s & 60 < t \end{cases}$$



- **Case 2**: sinusoidal reference

$$r(t) = 15 + 2\sin\left(\frac{2\pi}{60} t\right)$$

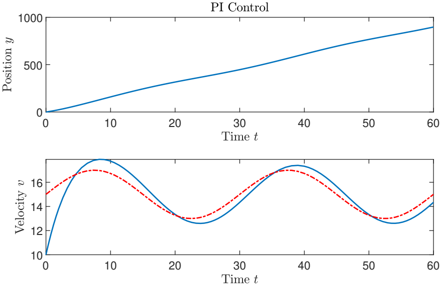


Dynamic Behavior Shaping with PI Control

- Parameters: $y_0 = 10$ m/s, $m = 500$ kg, $\delta = 0.5$, $\theta = 0°$
- Closed-loop PI control with $k_{\mathrm{p}} = 250$ and $k_{\mathrm{i}} = 50$

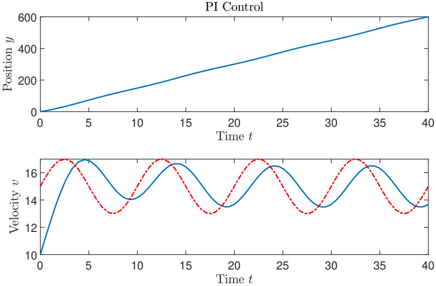
- **Case 3**: sinusoidal reference

$$r(t) = 15 + 2\sin\left(\frac{2\pi}{30}t\right)$$



- **Case 4**: sinusoidal reference

$$r(t) = 15 + 2\sin\left(\frac{2\pi}{10}t\right)$$



Dynamic Behavior Shaping with PI Control

Reference tracking: The same PI controller can make the closed-loop system follow a reference signal with small tracking error.

- To analyze the tracking behavior with respect to the frequency of the reference signal and to quantify the tracking error, we need to understand the system behavior in the Laplace domain
- The **bandwidth** of the closed-loop system provides an upper bound on the frequency of reference signals that can be tracked with small error

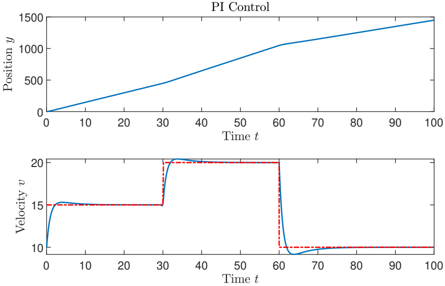
Robustness to Parameter Variations with PI Control

- Parameters: $y_0 = 10$ m/s, $m = 500$ kg, $\delta = 0.5$, $\theta = 0°$
- Closed-loop PI control with $k_\mathrm{p} = 250$ and $k_\mathrm{i} = 50$

- **Case 1**: mass change: $m = 200$ kg

  piecewise-constant reference

$$r(t) = \begin{cases} 15 m/s & t \leq 30 \\ 20 m/s & 30 < t \leq 60 \\ 10 m/s & 60 < t \end{cases}$$

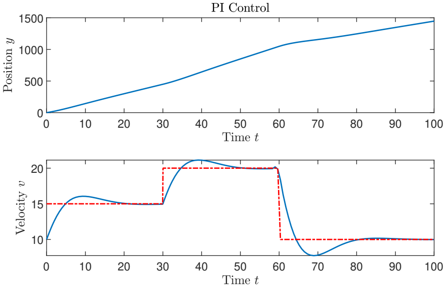


Robustness to Parameter Variations with PI Control

- Parameters: $y_0 = 10$ m/s, $m = 500$ kg, $\delta = 0.5$, $\theta = 0^\circ$
- Closed-loop PI control with $k_\mathrm{p} = 250$ and $k_\mathrm{i} = 50$
- **Case 2**: mass change: $m = 800$ kg

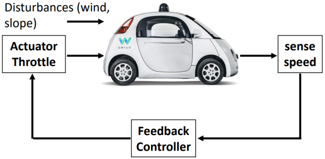
  piecewise-constant reference

$$r(t) = \begin{cases} 15 m/s & t \leq 30 \\ 20 m/s & 30 < t \leq 60 \\ 10 m/s & 60 < t \end{cases}$$



Robustness: The same PI controller can make the closed-loop system follow a reference signal even when some system parameters are not known exactly.

Summary



- Plant:

$$\dot{y}(t) = -\frac{\delta}{m} y(t) + \frac{1}{m} u(t) - g \sin(\theta)$$

- Error:

$$e(t) = r(t) - y(t)$$

- Controller:

$$u(t) = k_{\mathrm{p}} e(t) + k_{\mathrm{i}} \int_0^t e(t) dt + k_{\mathrm{d}} \frac{d}{dt} e(t)$$

Feedback control

- 1) *achieves reference signal tracking*,
- 2) *decreases* the sensitivity of the system response to parameter variations,
- 3) *attenuates* the effect of disturbances.