

Numerical ODE Solutions: Matlab ode45

- ▶ **Matlab ode45 function:**

$$[t, x] = \text{ode45}(\text{odefun}, \text{tspan}, x_0)$$

- ▶ **odefun:** function f defining the ode $\dot{x} = f(t, x)$
- ▶ **tspan:** time interval $[t_0, t_f]$
- ▶ **x0:** initial condition x_0
- ▶ **detailed description:**

<https://www.mathworks.com/help/matlab/ref/ode45.html>

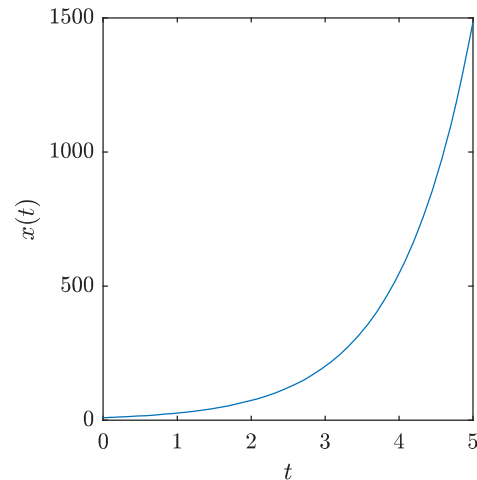
Example 1

- ▶ Consider the initial value problem:

$$\dot{x} = x, \quad x(0) = 10$$

- ▶ Determine the solution for $t \in [0, 5]$

```
%----- Example 1 -----  
% \dot x = x,  
% with x(0) = 10  
%-----  
  
f1 = @(t,x)(x); % vector field  
[ts,xs] = ode45(f1,[0,5],10);  
plot(ts,xs);
```



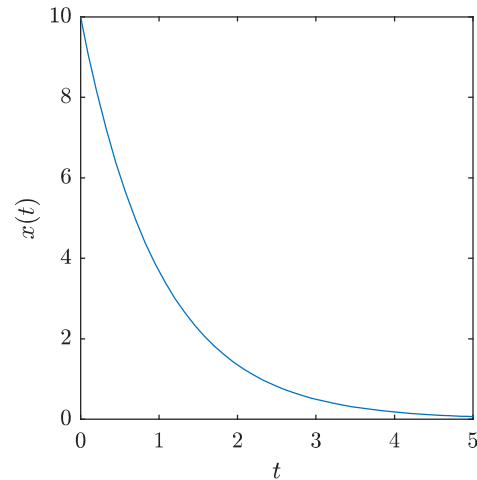
Example 2

- ▶ Consider the initial value problem:

$$\dot{x} = -x, \quad x(0) = 10$$

- ▶ Determine the solution for $t \in [0, 5]$

```
%----- Example 2 -----  
% \dot x = -x,  
% with x(0) = 10  
%-----  
f2      = @(t,x)(-x); % vector field  
[ts,xs] = ode45(f2,[0,5],10);  
plot(ts,xs);
```



Example 3

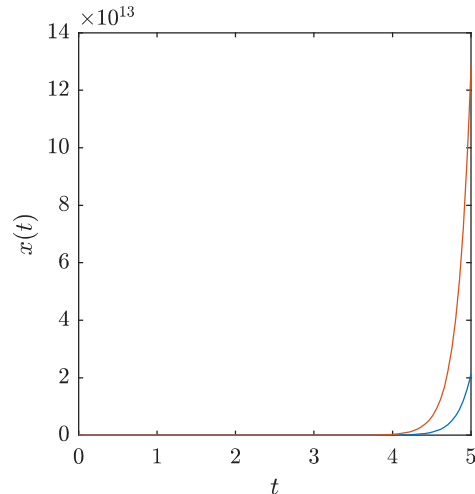
- ▶ Consider the initial value problem:

$$\ddot{z} - 3\dot{z} - 18z = 0, \quad z(0) = 3, \quad \dot{z}(0) = 9$$

- ▶ Determine the solution for $t \in [0, 5]$
- ▶ State-space model:
 - ▶ Variables: $x_1(t) = z(t)$ and $x_2(t) = \dot{z}(t)$
 - ▶ Model:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 18 & 3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, \quad \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 3 \\ 9 \end{bmatrix}$$

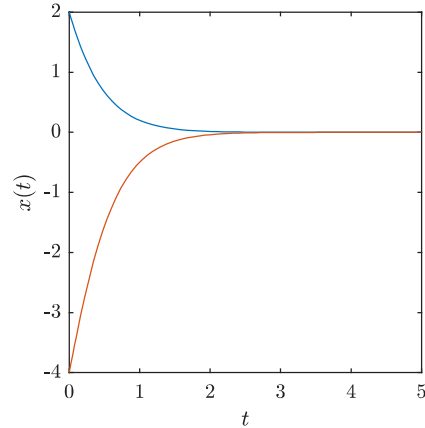
```
%----- Example 3 -----  
% \ddot{z} - 3 \dot{z} - 18 z = 0  
% with z(0) = 3, \dot{z}(0) = 9  
%-----  
[ts,xs] = ode45(@f3,[0,5],[3;9]);  
plot(ts,xs);  
  
function dx = f3(t,x)  
    dx = [0 1; 18 3]*x;  
end
```



Example 4 & 5

```
%----- Example 4 -----  
% \ddot{z} + 6 \dot{z} + 9z = 0  
% with z(0) = 2, \dot{z}(0) = -4  
%-----  
[ts,ys] = ode45(@f4,[0,5],10);
```

```
function dx = f4(t,x)  
    dotx = [0 1; -9 -6]*x;  
end
```



```
%----- Example 5 -----  
% \ddot{z} - 6 \dot{z} + 13 z = 0  
% with z(0) = 3, \dot{z}(0) = 17  
%-----  
[ts,ys] = ode45(@f5,[0,20],[3;17]);
```

```
function dx = f5(t,x)  
    dx = [0 1; -13 6]*x;  
end
```

