

ECE171A: Linear Control System Theory

Lecture 1: Introduction

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JACOBS SCHOOL OF ENGINEERING
Electrical and Computer Engineering

Outline

Course Overview

Control System Examples

Control System Modeling

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Control System Examples

Control System Modeling

Overview

- ▶ **ECE 171A: Linear Control System Theory** focuses on modeling, analysis, and design of single-input single-output linear time-invariant control systems with emphasis on frequency-domain techniques
- ▶ Topics:
 - ▶ **Modeling**: ordinary differential equations, linear time-invariant systems, first-order and second-order systems, block diagrams and signal flow graphs
 - ▶ **Analysis**: transient and steady-state behavior, Laplace transform, stability, root locus, frequency response, Bode plots, Nyquist plots, Nichols plots
 - ▶ **Design**: PID control, loop shaping

Prerequisites

- ▶ Prerequisites
 - ▶ **Introductory physics:** Newton's law, Ohm's law, Kirchhoff's voltage and current laws
 - ▶ **Calculus:** derivatives, integration, exponential function, Taylor series
 - ▶ **Programming experience** with MATLAB, Python, or similar language
 - ▶ Optional but helpful: ordinary differential equations, linear algebra
- ▶ Courses that fulfill these prerequisites: **ECE 45** or **MAE 40**

Teaching Team



- ▶ Instructor:
 - ▶ Nikolay Atanasov
 - ▶ Assistant Professor, ECE Department
 - ▶ Email: natanasov@ucsd.edu



- ▶ Teaching Assistant:
 - ▶ Alakh Desai
 - ▶ PhD Student, ECE Department
 - ▶ Email: ahdesai@ucsd.edu

Discussion Session and Office Hours

▶ Discussion Session

- ▶ Mondays, 3:00 pm - 3:50 pm, in PCYNH 121
- ▶ Optional material and examples supplementing the lectures
- ▶ Please ask questions and start a discussion!

▶ Office Hours

- ▶ Wednesdays, 1:00 pm - 2:00 pm on Zoom (see Canvas for information)
 - ▶ No new material will be covered
 - ▶ The focus will be on answering your questions and helping you with the material
- ▶ Additional office hours by appointment

Please try to attend the discussion and office hours. Even if you do not have questions, you can meet new people, help answer questions, and create a supportive community for this class.

Assignments and Grading

- ▶ Course website: <https://natanaso.github.io/ece171a>
- ▶ Includes links to:
 - ▶ **Canvas**: course password, lecture recordings
 - ▶ **Gradescope**: homework submission and grades
 - ▶ **Piazza**: discussion and class announcements (**please check regularly!**)
- ▶ Assignments:
 - ▶ Academic integrity quiz on Canvas (**due by Oct 12**)
 - ▶ 6 homework sets (35% of grade)
 - ▶ Project (10% of grade)
 - ▶ Midterm exam (25% of grade): calculator + single-sided cheat sheet
 - ▶ Final exam (30% of grade): calculator + double-sided cheat sheet
- ▶ Submission:
 - ▶ The assignment release dates are shown on the website
 - ▶ The assignment due dates are stated in each assignment
 - ▶ Homework sets typically due in 1 week
 - ▶ **No late submissions**: homework submitted past the deadline will not be accepted without serious justification
- ▶ Grading:
 - ▶ Rubric: 85+: A; 80-85: A-; 75-80: B+; 65-75: B; 60-65: B-; 55-60: C+; ...
 - ▶ The rubric may be adjusted at the instructor's discretion

References

- ▶ **Primary Textbook:**

Karl J. Åström and Richard M. Murray, *Feedback Systems: An Introduction for Scientists and Engineers*, 2008

Available at: <https://fbswiki.org/>

- ▶ **Additional References:** listed on the course website (not required)

Course Schedule (Tentative)

Week	Date	Lecture	Material	Assignment
1	Oct 02	Introduction	Ch. 1	
	Oct 04	Feedback Principles	Ch. 2	
2	Oct 09	Discussion		
	Oct 09	System Modeling	Ch. 3	
	Oct 11	Solving ODEs	Ch. 5	HW1
3	Oct 16	Discussion		
	Oct 16	Laplace Transform, Transfer Function	Ch. 9	
	Oct 18	Catch-up		HW2
4	Oct 23	Discussion]		
	Oct 23	Block Diagram, Signal Flow Graph		
	Oct 25	Stability, Routh-Hurwitz	Ch. 6	HW3
5	Oct 30	Discussion		
	Oct 30	Catch-up		
	Nov 01	Transient and Steady-state Response	Ch. 6	
6	Nov 06	Discussion		
	Nov 06	Midterm Exam		

Check the course website for reading material and schedule updates:
<https://natanaso.github.io/ece171a>

Outline

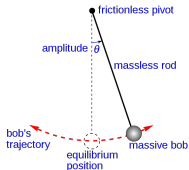
Course Overview

Control System Examples

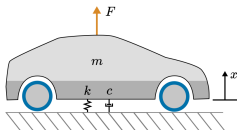
Control System Modeling

What is a dynamical system?

- ▶ A **dynamical system** is a system whose behavior changes over **time**, often in response to internal or external stimulation



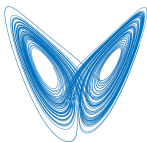
(a) Pendulum



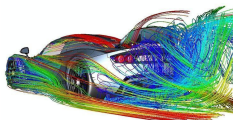
(b) Car suspension



(c) Water flow



(d) Atmospheric convection
(Lorenz system)



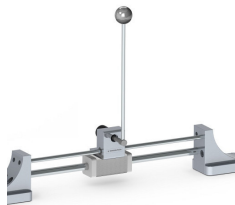
(e) Fluid dynamics



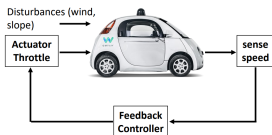
(f) Sync of fish

What is a control system?

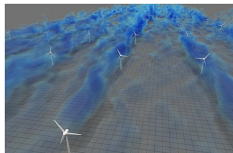
- ▶ A **control system** is an interconnection of components (dynamical systems) that provides a desired response
- ▶ A **controller** is a component of a control system that modifies the overall system behavior



(a) Inverted pendulum



(b) Cruise control



(c) Wind farm

Control System

- ▶ Modern control systems include physical and cyber components
- ▶ A **physical component** is a mechanical, electrical, fluid, or thermal device acting as a sensor, actuator, or embedded system component:
 - ▶ A **sensor** is a device that provides measurements of a signal of interest (*system output*)
 - ▶ An **actuator** is a device that alters the configuration of the system or its environment (*system input*)
- ▶ A **cyber component** is a software element that executes a specific function
- ▶ **Control system engineering** focuses on:
 - ▶ modeling cyber-physical systems,
 - ▶ analyzing system behavior,
 - ▶ designing control techniques to achieve desired behavior
- ▶ **Performance characteristics**: stability, transient and steady-state tracking, rejection of external disturbances, robustness to modeling uncertainties, etc.

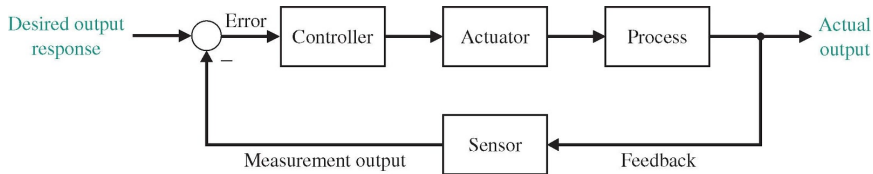
Open-Loop vs Closed-Loop Control Systems

- ▶ An **open-loop (feedforward) control system** utilizes a controller without measurement feedback of the system output



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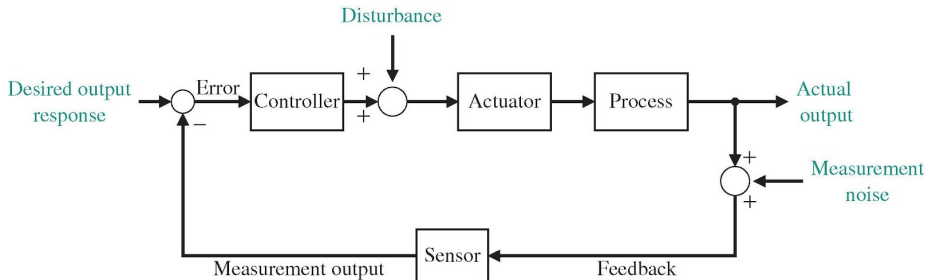
- ▶ A **closed-loop (feedback) control system** utilizes a controller with measurement feedback of the system output



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Noise and Modeling Errors

- ▶ A closed-loop control system uses measurement feedback to compute and reduce the **error** between the desired and measured output
- ▶ Closed-loop control attenuates the effects of **process noise** (disturbance), **measurement noise**, and **modeling errors**

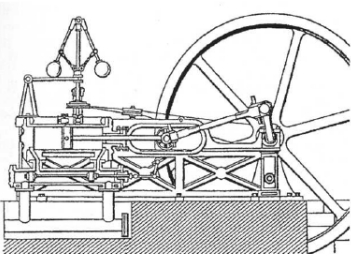
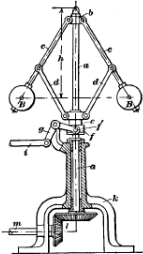
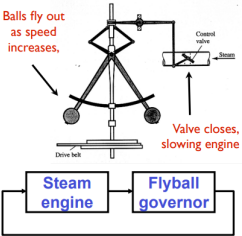


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Feedback Control Examples

Flyball Governor (1788)

- ▶ regulate speed of a steam engine
- ▶ reduces the effect of load variations
- ▶ major advance of industrial revolution



Feedback Control Examples



(a) Drones



(b) Autonomous vehicles

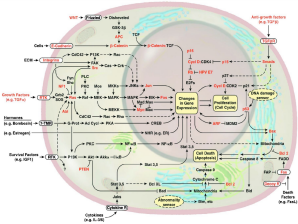


(c) Rockets

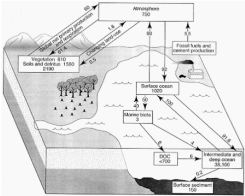


(d) Aircraft

Feedback Control Examples



(e) Biological systems



(f) Environmental systems



(g) Social networks

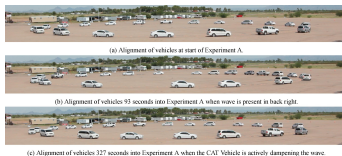


(h) Finance market

Feedback Control Examples



(a) Robot parkour



(b) Traffic wave control



(c) Autonomous flight

Outline

Course Overview

Control System Examples

Control System Modeling

Control System Modeling

- ▶ A mathematical model is a key element in the design and analysis of control systems
- ▶ Dynamic behavior is described by **ordinary differential equations** (ODEs)

$$\frac{d}{dt}y(t) + a(t)y(t) = u(t)$$

- ▶ The relationship between the variables and their derivatives in an ODE may be **linear** or **nonlinear**
- ▶ Nonlinear ODEs are often approximated using **linearization** because linear ODEs are much easier to analyze
- ▶ The coefficients of an ODE may be **time-invariant** or **time-varying**
- ▶ This class will focus on **linear time-invariant** (LTI) ODE systems

Differential Equations of Physical Systems

- ▶ Physical systems from different domains (electrical, mechanical, fluid, thermal) contain elements that share similar roles

Variable	Electrical	Mechanical	Fluid	Thermal
Through	Current	Force, Torque	Flow rate	Flow rate
Across	Voltage	Velocity	Pressure	Temperature
Inductive	Inductance	Inverse Stiffness	Inertia	–
Capacitive	Capacitance	Mass, Moment of Inertia	Capacitance	Capacitance
Resistive	Resistance	Friction	Resistance	Resistance

- ▶ The dynamic behavior of these elements is described by physical laws, such as Kirchhoff's laws or Newton's laws, enabling an ODE description of the system

Through and Across Element Variables




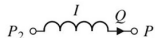
Table 2.1 Summary of Through- and Across-Variables for Physical Systems

System	Variable Through Element	Integrated Through-Variable	Variable Across Element	Integrated Across-Variable
Electrical	Current, i	Charge, q	Voltage difference, v_{21}	Flux linkage, λ_{21}
Mechanical translational	Force, F	Translational momentum, P	Velocity difference, v_{21}	Displacement difference, y_{21}
Mechanical rotational	Torque, T	Angular momentum, h	Angular velocity difference, ω_{21}	Angular displacement difference, θ_{21}
Fluid	Fluid volumetric rate of flow, Q	Volume, V	Pressure difference, P_{21}	Pressure momentum, γ_{21}
Thermal	Heat flow rate, q	Heat energy, H	Temperature difference, \mathcal{T}_{21}	

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Inductive Elements



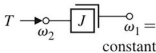
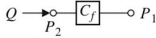
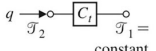
Table 2.2 Summary of Governing Differential Equations for Ideal Elements

Type of Element	Physical Element	Governing Equation	Energy E or Power \mathcal{P}	Symbol
Inductive storage	Electrical inductance	$v_{21} = L \frac{di}{dt}$	$E = \frac{1}{2} Li^2$	
	Translational spring	$v_{21} = \frac{1}{k} \frac{dF}{dt}$	$E = \frac{1}{2} \frac{F^2}{k}$	
	Rotational spring	$\omega_{21} = \frac{1}{k} \frac{dT}{dt}$	$E = \frac{1}{2} \frac{T^2}{k}$	
	Fluid inertia	$P_{21} = I \frac{dQ}{dt}$	$E = \frac{1}{2} IQ^2$	

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Capacitive Elements

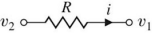
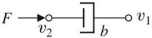
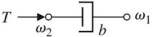
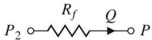
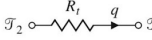
Table 2.2 Summary of Governing Differential Equations for Ideal Elements

Type of Element	Physical Element	Governing Equation	Energy E or Power \mathcal{P}	Symbol
Capacitive storage	Electrical capacitance	$i = C \frac{dv_{21}}{dt}$	$E = \frac{1}{2} C v_{21}^2$	
	Translational mass	$F = M \frac{dv_2}{dt}$	$E = \frac{1}{2} M v_2^2$	
	Rotational mass	$T = J \frac{d\omega_2}{dt}$	$E = \frac{1}{2} J \omega_2^2$	
	Fluid capacitance	$Q = C_f \frac{dP_{21}}{dt}$	$E = \frac{1}{2} C_f P_{21}^2$	
	Thermal capacitance	$q = C_t \frac{d\mathcal{T}_2}{dt}$	$E = C_t \mathcal{T}_2$	

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Resistive Elements

Table 2.2 Summary of Governing Differential Equations for Ideal Elements

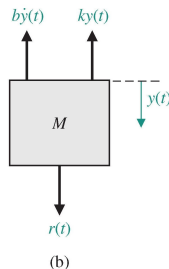
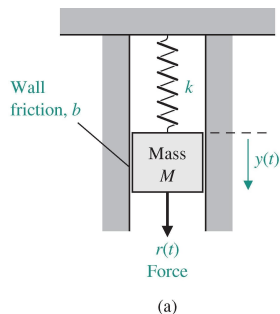
Type of Element	Physical Element	Governing Equation	Energy E or Power \mathcal{P}	Symbol
Energy dissipators	Electrical resistance	$i = \frac{1}{R}v_{21}$	$\mathcal{P} = \frac{1}{R}v_{21}^2$	
	Translational damper	$F = bv_{21}$	$\mathcal{P} = bv_{21}^2$	
	Rotational damper	$T = b\omega_{21}$	$\mathcal{P} = b\omega_{21}^2$	
	Fluid resistance	$Q = \frac{1}{R_f}P_{21}$	$\mathcal{P} = \frac{1}{R_f}P_{21}^2$	
	Thermal resistance	$q = \frac{1}{R_t}\mathcal{T}_{21}$	$\mathcal{P} = \frac{1}{R_t}\mathcal{T}_{21}$	

Spring-Mass-Damper Example

- ▶ The behavior of a spring-mass-damper system is described by Newton's second law:

$$M \frac{d^2 y(t)}{dt^2} + \underbrace{b \frac{dy(t)}{dt}}_{\text{viscous damper}} + \underbrace{ky(t)}_{\text{spring force}} = \underbrace{r(t)}_{\text{input force}}$$

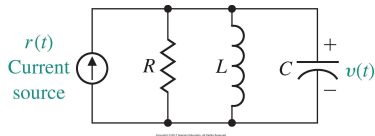
- ▶ The mass displacement $y(t)$ satisfies a second-order linear time-invariant (LTI) ordinary differential equation (ODE)



Parallel RLC Circuit Example

- ▶ The behavior of an electrical RLC circuit is described by Kirchhoff's current law:

$$r(t) = i_R(t) + i_L(t) + i_C(t)$$



- ▶ Parallel devices have the same voltage $v(t)$:

- ▶ Resistor: $v(t) = Ri_R(t)$
- ▶ Inductor: $v(t) = L \frac{di_L(t)}{dt}$
- ▶ Capacitor: $i_C(t) = C \frac{dv(t)}{dt}$

- ▶ The inductor current $i_L(t)$ satisfies a second-order LTI ODE:

$$CL \frac{d^2 i_L(t)}{dt^2} + \frac{L}{R} \frac{di_L(t)}{dt} + i_L(t) = r(t)$$

Laplace Transform

- ▶ The behavior of an LTI ODE system may be studied in the **time domain** ($t \in \mathbb{R}$) or in the **complex (frequency) domain** ($s = \sigma + j\omega \in \mathbb{C}$)
- ▶ The **Laplace transform** converts an **LTI ODE** in the time domain into a **linear algebraic equation** in the complex domain
- ▶ Example:
 - ▶ Time domain LTI ODE:
 - ▶ Laplace domain linear algebraic eq.:

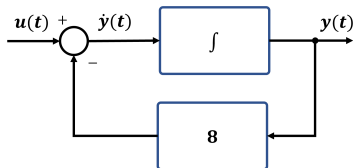
$$\frac{d}{dt}y(t) + 8y(t) = u(t)$$

$$sY(s) - y(0) + 8Y(s) = U(s)$$

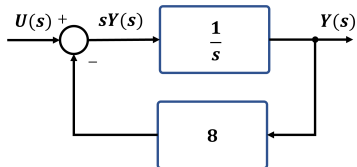
Block Diagram

- ▶ The system components may be visualized as a **block diagram**
- ▶ A **block** represents the input-output relationship of a system component
- ▶ To represent multi-component systems, the blocks are interconnected
- ▶ Signals may be added or subtracted using **summing points**
- ▶ A block diagram may represent a system in the time domain or in the complex domain

- ▶ Time domain:

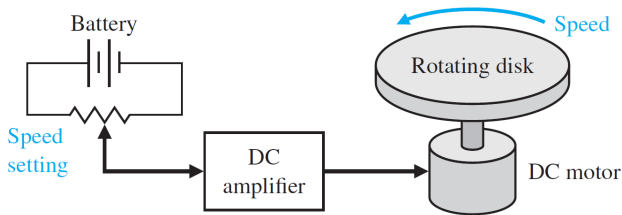


- ▶ Laplace domain:



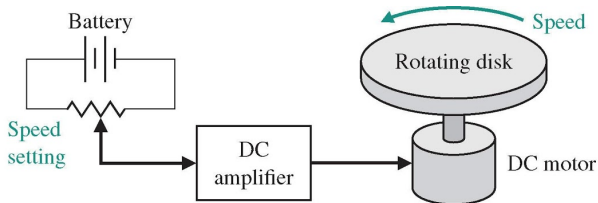
Example: Rotating Disk Speed Control

- ▶ Line-cell imaging in biomedical applications uses spinning disk conformal microscope
- ▶ Objective: design a controller for a rotating disk system to ensure the speed of rotation is within a specified percentage of a desired speed
- ▶ System components:
 - ▶ DC motor: provides speed proportional to the applied voltage
 - ▶ Battery source: provides voltage proportional to the desired speed
 - ▶ DC amplifier: amplifies the battery voltage to meet the motor voltage requirements
 - ▶ Tachometer: provides output voltage proportional to the speed of its shaft

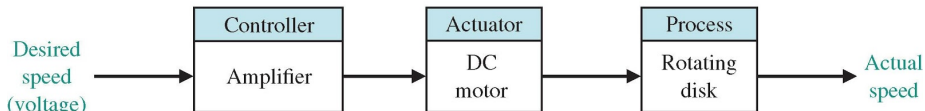


(a)

Open-Loop Rotating Disk System



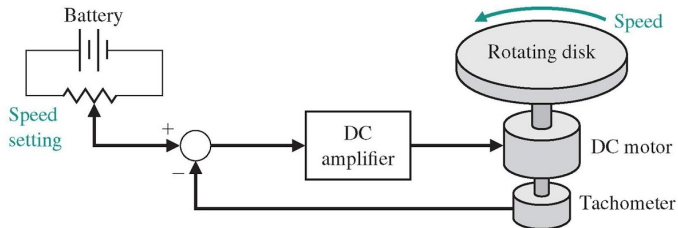
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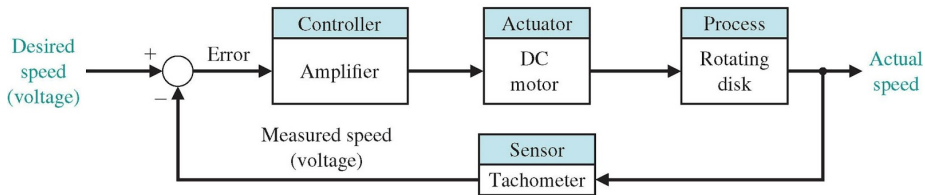
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Closed-Loop Rotating Disk System



(a)

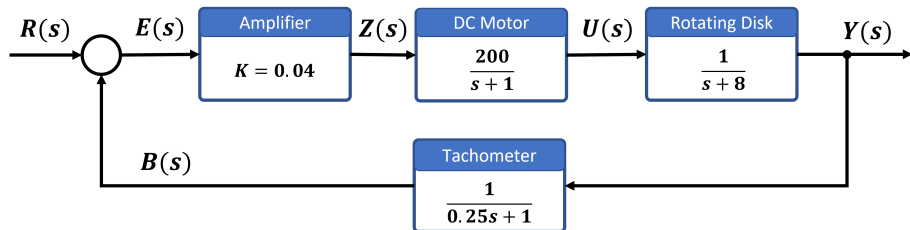


(b)

Control System Analysis

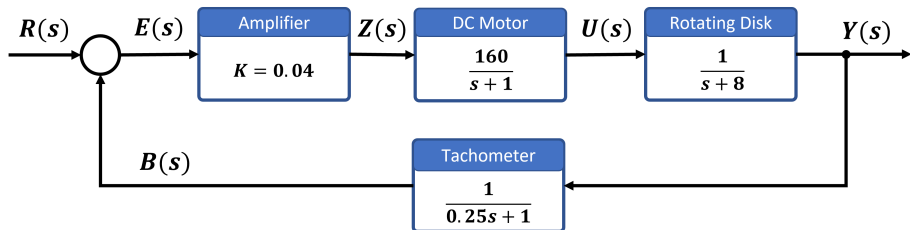
- ▶ The system components are described using LTI ODEs
- ▶ Time domain:
 - ▶ Desired speed: $r(t)$
 - ▶ Amplifier: $z(t) = Kr(t)$
 - ▶ DC motor: $\dot{u}(t) + u(t) = 200z(t)$
 - ▶ Rotating disk: $\dot{y}(t) + 8y(t) = u(t)$
 - ▶ Tachometer: $\dot{b}(t) + 4b(t) = 4y(t)$
- ▶ Laplace domain:
 - ▶ Desired speed: $R(s)$
 - ▶ Amplifier: $Z(s) = KR(s)$
 - ▶ DC motor: $U(s) = \frac{200}{s+1}Z(s)$
 - ▶ Rotating disk: $Y(s) = \frac{1}{s+8}U(s)$
 - ▶ Tachometer: $B(s) = \frac{4}{s+4}Y(s)$
- ▶ We will study how to choose the amplifier gain K to ensure that system output $y(t)$ tracks a desired reference signal $r(t)$

Nominal Rotating Disk System



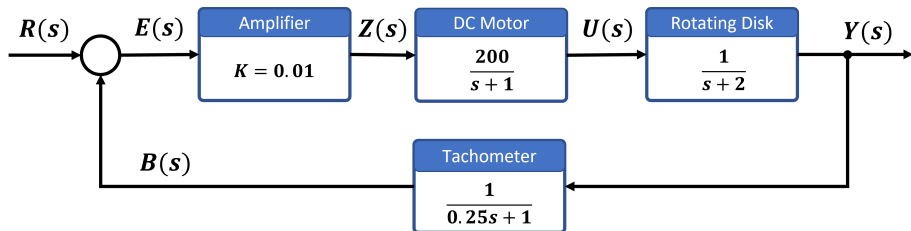
- ▶ A nominal model aims to capture the system behavior accurately but parameter errors or disturbances might be present
- ▶ Closed-loop control becomes important when there are parameter errors and disturbances

Low-gain Rotating Disk System



- ▶ The DC motor gain might be different in the real system (e.g., 160) compared to the nominal model (e.g., 200)

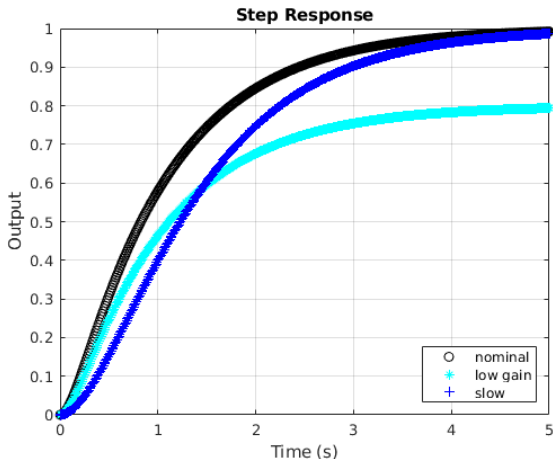
Slow Rotating Disk System



- ▶ The disk might rotate slower in the real system (e.g., $\dot{y}(t) + 2y(t) = u(t)$) compared to the nominal model (e.g., $\dot{y}(t) + 8y(t) = u(t)$)

Open-Loop Step Response

- ▶ Without feedback, the real system response might be different than what was planned



Closed-Loop Step Response

- ▶ Feedback improves the sensitivity to parameter errors and disturbances
- ▶ Despite the advantages, feedback architectures need to be designed carefully to avoid oscillations and steady-state error

