ECE276B: Planning & Learning in Robotics
Lecture 9: Sampling-Based Motion Planning

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Outline

Motion Planning Problem

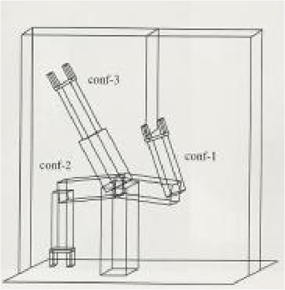
- Configuration space: C; Obstacle space: C_{obs}; Free space: C_{free}
- Start state: $\mathbf{x}_s \in C_{free}$; Goal state: $\mathbf{x}_\tau \in C_{free}$
- Path: continuous function $\rho : [0,1] \to C$; Set of all paths: $\mathcal{P}$
- Feasible path: continuous function $\rho : [0,1] \to C_{free}$ such that $\rho(0) = \mathbf{x}_s$ and $\rho(1) = \mathbf{x}_\tau$; Set of all feasible paths: $\mathcal{P}_{s,\tau}$
- **Motion planning problem**: Given free space C_{free}, obstacle space C_{obs}, start state $\mathbf{x}_s \in C_{free}$, goal state $\mathbf{x}_\tau \in C_{free}$, and cost function $J : \mathcal{P} \to \mathbb{R}_{\geq 0}$, find a feasible path ρ^* such that:

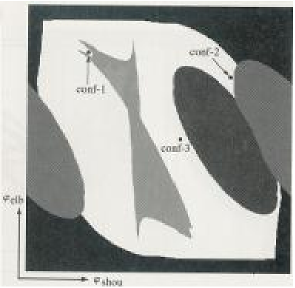
$$J(\rho^*) = \min_{\rho \in \mathcal{P}_{s,\tau}} J(\rho)$$

  or report failure if no such path exists

Search-Based Planning

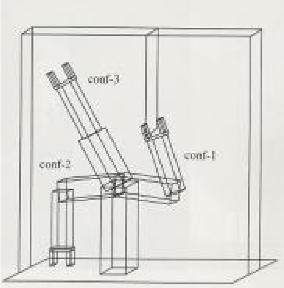
- Generates a graph by systematic discretization of C_{free}
- Searches the graph for a feasible path, guaranteeing to find one if it exists (**resolution complete**)
- Provides finite-time suboptimality bounds on the solution
- Can interleave graph construction and search, i.e., nodes added only when necessary
- Computationally expensive in high dimensional configuration spaces

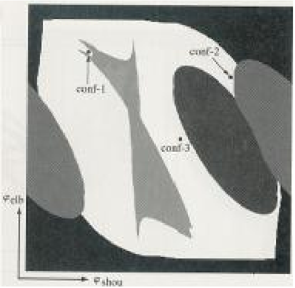


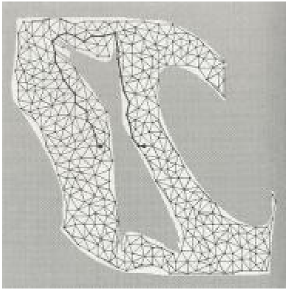


Sampling-Based Planning

- Generates a graph by random sampling in C_{free}
- Searches the graph for a path, guaranteeing that the probability of finding one, if it exists, approaches 1 as the number of iterations $\rightarrow \infty$ (**probabilistically complete**)
- Provides asymptotic suboptimality bounds on the solution
- Can interleave graph construction and search, i.e., samples added only when necessary
- Requires less memory than search-based planning in high dimensions







Functions for Sampling-Based Motion Planning

- SAMPLE: returns independent identically distributed (iid) samples from C
- SAMPLEFREE: returns iid samples from C_{free}
- NEAREST: given a graph $G = (V, E)$ with $V \subset C$ and a point $\mathbf{x} \in C$, returns a vertex $\mathbf{v} \in V$ that is closest to $\mathbf{x}$:
$$\textsc{Nearest}((V,E),\mathbf{x}) := \arg\min_{\mathbf{v}\in V} \|\mathbf{x}-\mathbf{v}\|$$
- NEAR: given a graph $G = (V, E)$ with $V \subset C$, a point $\mathbf{x} \in C$, and $r > 0$, returns the vertices in V that are within a distance r from $\mathbf{x}$:
$$\textsc{Near}((V,E),\mathbf{x},r) := \{\mathbf{v} \in V \mid \|\mathbf{x}-\mathbf{v}\| \leq r\}$$
- $\textsc{Steer}_\epsilon$: given points $\mathbf{x}, \mathbf{y} \in C$ and $\epsilon > 0$, returns a point $\mathbf{z} \in C$ that minimizes $\|\mathbf{z}-\mathbf{y}\|$ while remaining within ϵ from $\mathbf{x}$:
$$\textsc{Steer}_\epsilon(\mathbf{x},\mathbf{y}) := \arg\min_{\mathbf{z}:\|\mathbf{z}-\mathbf{x}\|\leq\epsilon} \|\mathbf{z}-\mathbf{y}\| = \mathbf{x} + \epsilon\frac{\mathbf{y}-\mathbf{x}}{\|\mathbf{y}-\mathbf{x}\|}$$
- COLLISIONFREE: given points $\mathbf{x}, \mathbf{y} \in C$, returns TRUE if the line segment between $\mathbf{x}$ and $\mathbf{y}$ lies in C_{free} and FALSE otherwise.

Outline

Search-Based vs Sampling-Based Planning

Probabilistic Roadmap

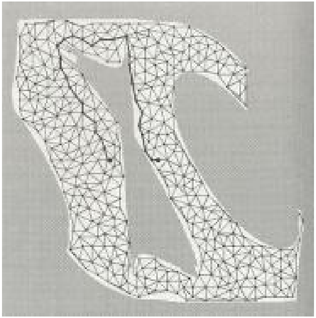
Rapidly Exploring Random Tree

RRT*

Probabilistic Roadmap (PRM)

Step 1: **Construction Phase**: Build a graph G (**roadmap**) aiming to make it accessible from any point in C_{free}

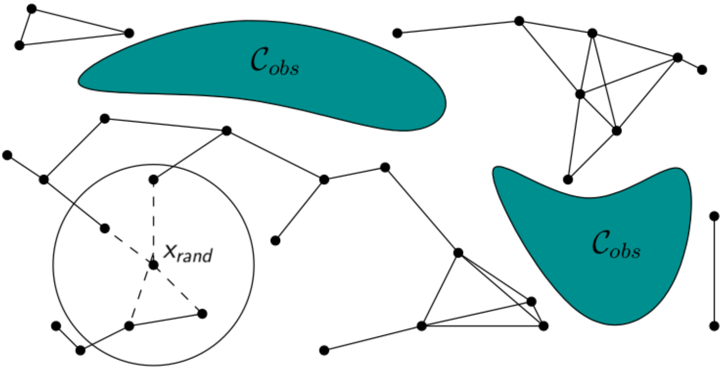
- **Nodes**: randomly sampled valid configurations in C_{free}
- **Edges**: added between samples that are easy to connect with local control (e.g., follow straight line)



Step 2: **Query Phase**: Given start $\mathbf{x}_s \in C_{free}$ and goal $\mathbf{x}_\tau \in C_{free}$, connect them to the graph G and search G for a shortest path from $\mathbf{x}_s$ to $\mathbf{x}_\tau$

- **Pros and Cons**:
  - Simple and effective in high dimensional configuration spaces
  - Has difficulties with narrow passages
  - Can result in suboptimal paths; only asymptotic guarantees on optimality
  - Enables **multi-query planning**: different start and goal configurations in the same environment

Step 1: PRM Construction Phase



Step 1: PRM Construction Phase

Algorithm PRM (construction phase)

$V \leftarrow \emptyset$; $E \leftarrow \emptyset$
for $i = 1, \ldots, n$ **do**
  $\mathbf{x}_{rand} \leftarrow \textsc{SampleFree}()$
  $V \leftarrow V \cup \{\mathbf{x}_{rand}\}$
  for $\mathbf{x} \in \textsc{Near}((V, E), \mathbf{x}_{rand}, r)$ **do** ▷ May use k nearest nodes
    if (**not** G.same_component($\mathbf{x}_{rand}, \mathbf{x}$)) **and** $\textsc{CollisionFree}(\mathbf{x}_{rand}, \mathbf{x})$ **then**
      $E \leftarrow E \cup \{(\mathbf{x}_{rand}, \mathbf{x}), (\mathbf{x}, \mathbf{x}_{rand})\}$
return $G = (V, E)$

- G.same_component($\mathbf{x}_{rand}, \mathbf{x}$)
  - ensures that $\mathbf{x}$ and $\mathbf{x}_{rand}$ are in different connected components of G
  - every connection decreases the number of connected components in G
  - efficient implementation using a union-find algorithm
  - may be replaced by G.vertex_degree($\mathbf{x}$) $< K$ for some fixed K (e.g., $K = 15$) if it is important to generate multiple alternative paths

Asymptotically Optimal Probabilistic Roadmap

- To achieve an asymptotically optimal PRM, the connection radius r should decrease such that the average number of connections attempted from a roadmap vertex is proportional to $log(n)$:

$$r^* > 2\left(1+\frac{1}{d}\right)^{1/d}\left(\frac{Vol(C_{free})}{Vol(\text{Unit d-ball})}\right)^{1/d}\left(\frac{\log(n)}{n}\right)^{1/d}$$

- S. Karaman and E. Frazzoli, "Incremental Sampling-based Algorithms for Optimal Motion Planning," IJRR, 2010

Algorithm PRM* (construction phase)

$V \leftarrow \{\mathbf{x}_s\} \cup \{\textsc{SampleFree}()\}_{i=1}^{n}$; $E \leftarrow \emptyset$
for $\mathbf{v} \in V$ **do**
  for $\mathbf{x} \in \textsc{Near}((V, E), \mathbf{v}, r^*) \setminus \{\mathbf{v}\}$ **do**
    if $\textsc{CollisionFree}(\mathbf{v}, \mathbf{x})$ **then**
      $E \leftarrow E \cup \{(\mathbf{v}, \mathbf{x}), (\mathbf{x}, \mathbf{v})\}$
return $G = (V, E)$

Outline

Search-Based vs Sampling-Based Planning

Probabilistic Roadmap

Rapidly Exploring Random Tree

RRT*

Rapidly Exploring Random Tree (RRT)

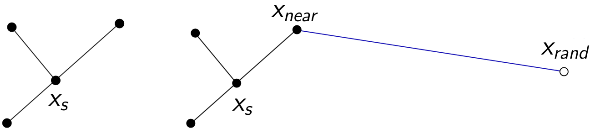
- **Rapidly Exploring Random Tree** (RRT):
  - Steven LaValle, "Rapidly-exploring random trees: A new tool for path planning," Technical Report, Iowa State University, October 1998
  - One of the most popular planning techniques
  - Many, many, many extensions and variants (articulated robots, kinematics, dynamics, differential constraints)
  - There exist incremental versions of RRTs that reuse a previously constructed tree when replanning in response to map updates

- **PRM**:
  - graph constructed from random samples that can be searched for a path whenever a start node $\mathbf{x}_s$ and goal node $\mathbf{x}_\tau$ are specified
  - well-suited for repeated planning between different pairs of $\mathbf{x}_s$ and $\mathbf{x}_\tau$ (**multi-query planning**)

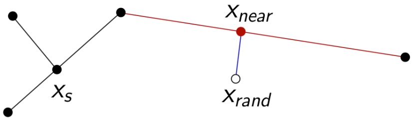
- **RRT**:
  - tree constructed from random samples with root $\mathbf{x}_s$ and grown until $\mathbf{x}_\tau$ is contained
  - well-suited for single-shot planning between a fixed pair of $\mathbf{x}_s$ and $\mathbf{x}_\tau$ (**single-query planning**)

Rapidly Exploring Random Tree (RRT)

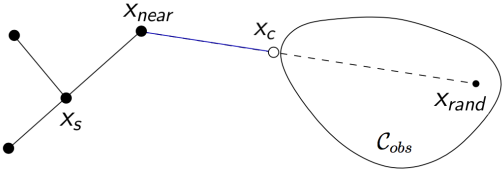
- **Construction Phase**: sample a new configuration $\mathbf{x}_{rand} \in C_{free}$, find the nearest neighbor $\mathbf{x}_{nearest}$ in G, connect them if straight line is collision-free:



- (Variant) if $\mathbf{x}_{nearest}$ lies on an existing edge, then split the edge:



- (Variant) if there is an obstacle, travel up to the obstacle boundary as far as allowed by a collision detection algorithm

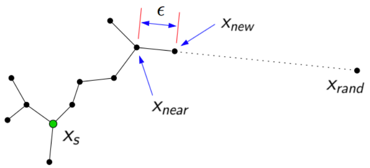


Rapidly Exploring Random Tree (RRT)

- Starting with an initial configuration $\mathbf{x}_s$ build a tree until the goal configuration $\mathbf{x}_\tau$ is part of it

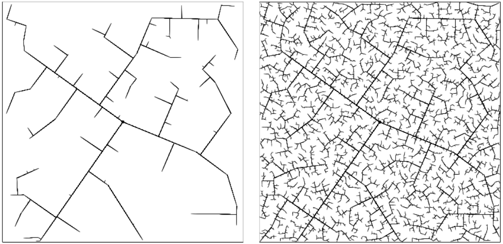
Algorithm RRT

1: $V \leftarrow \{\mathbf{x}_s\}$; $E \leftarrow \emptyset$
2: **for** $i = 1 \ldots n$ **do**
3: $\quad \mathbf{x}_{rand} \leftarrow \textsc{SampleFree}()$
4: $\quad \mathbf{x}_{nearest} \leftarrow \textsc{Nearest}((V, E), \mathbf{x}_{rand})$
5: $\quad \mathbf{x}_{new} \leftarrow \textsc{Steer}_\epsilon(\mathbf{x}_{nearest}, \mathbf{x}_{rand})$
6: $\quad$ **if** $\textsc{CollisionFree}(\mathbf{x}_{nearest}, \mathbf{x}_{new})$ **then**
7: $\qquad V \leftarrow V \cup \{\mathbf{x}_{new}\}$; $E \leftarrow E \cup \{(\mathbf{x}_{nearest}, \mathbf{x}_{new})\}$
8: **return** $G = (V, E)$



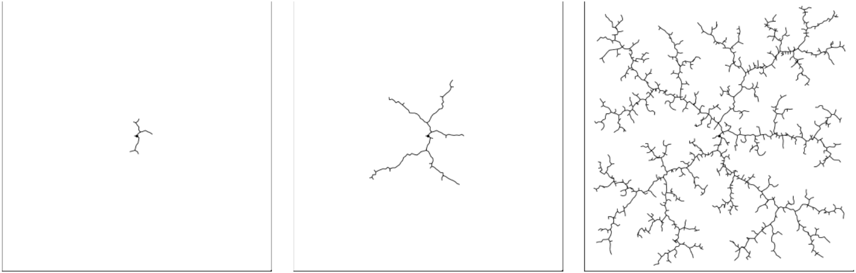
Rapidly Exploring Random Tree (RRT)

- RRT with $\epsilon = \infty$ (called Rapidly Exploring Dense Tree (RDT)):



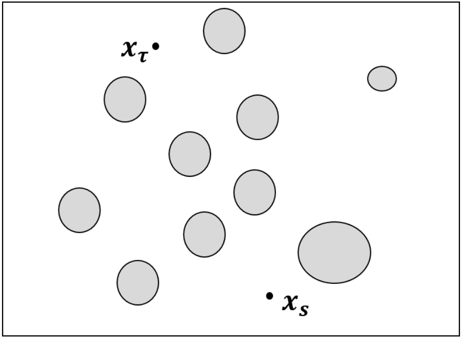
45 iterations   2345 iterations

- RRT with $\epsilon < \infty$



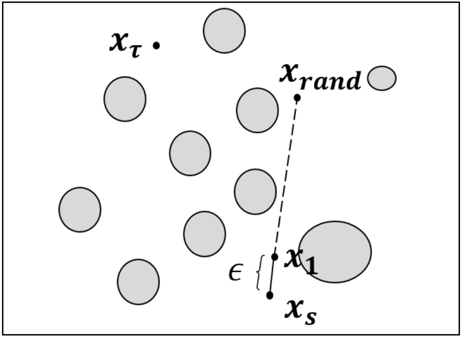
Example: RRT Algorithm

- Start node $\mathbf{x}_s$
- Goal node $\mathbf{x}_\tau$
- Gray obstacles



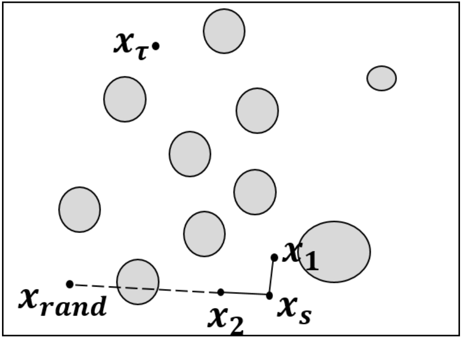
Example: RRT Algorithm

- Sample $\mathbf{x}_{rand} \in C_{free}$
- Steer from $\mathbf{x}_s$ towards $\mathbf{x}_{rand}$ by a fixed distance ϵ to get $\mathbf{x}_1$
- If the segment from $\mathbf{x}_s$ to $\mathbf{x}_1$ is collision-free, insert $\mathbf{x}_1$ into the tree



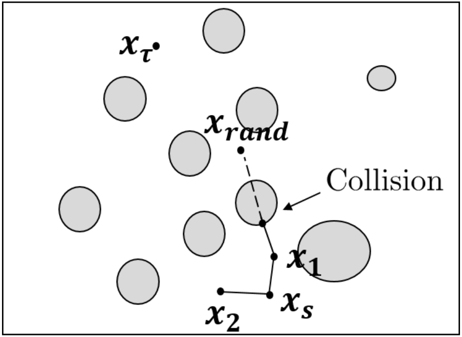
Example: RRT Algorithm

- Sample $\mathbf{x}_{rand} \in C_{free}$
- Find the closest node $\mathbf{x}_{nearest}$ to $\mathbf{x}_{rand}$
- Steer from $\mathbf{x}_{nearest}$ towards $\mathbf{x}_{rand}$ by a fixed distance ϵ to get $\mathbf{x}_2$
- If the segment from $\mathbf{x}_{nearest}$ to $\mathbf{x}_2$ is collision-free, insert $\mathbf{x}_2$ into the tree



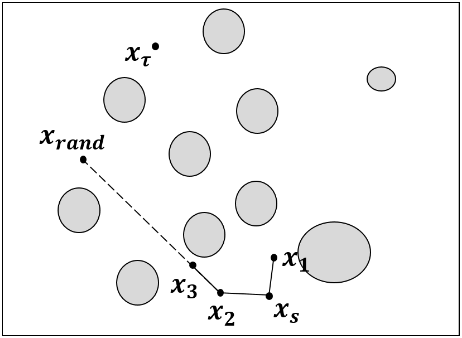
Example: RRT Algorithm

- Sample $\mathbf{x}_{rand} \in C_{free}$
- Find the closest node $\mathbf{x}_{nearest}$ to $\mathbf{x}_{rand}$
- Steer from $\mathbf{x}_{nearest}$ towards $\mathbf{x}_{rand}$ by a fixed distance ϵ to get $\mathbf{x}_3$
- If the segment from $\mathbf{x}_{nearest}$ to $\mathbf{x}_3$ is collision-free, insert $\mathbf{x}_3$ into the tree



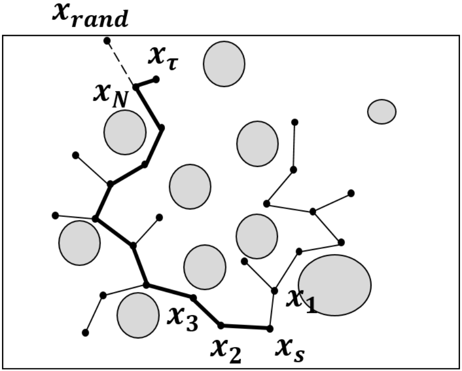
Example: RRT Algorithm

- Sample $\mathbf{x}_{rand} \in C_{free}$
- Find the closest node $\mathbf{x}_{nearest}$ to $\mathbf{x}_{rand}$
- Steer from $\mathbf{x}_{nearest}$ towards $\mathbf{x}_{rand}$ by a fixed distance ϵ to get $\mathbf{x}_3$
- If the segment from $\mathbf{x}_{nearest}$ to $\mathbf{x}_3$ is collision-free, insert $\mathbf{x}_3$ into the tree



Example: RRT Algorithm

- Continue until a node that is a distance ϵ from the goal is generated
- Either terminate the algorithm or search for additional feasible paths

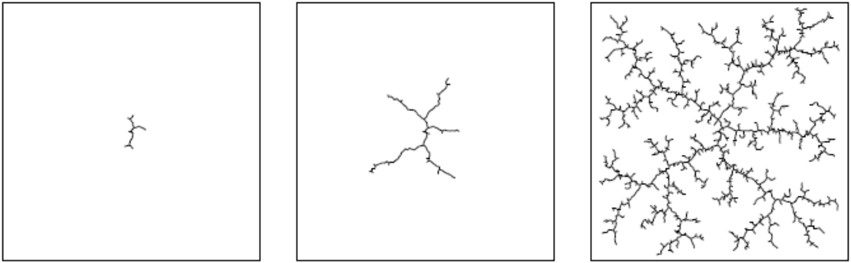


RRT Implementation Details

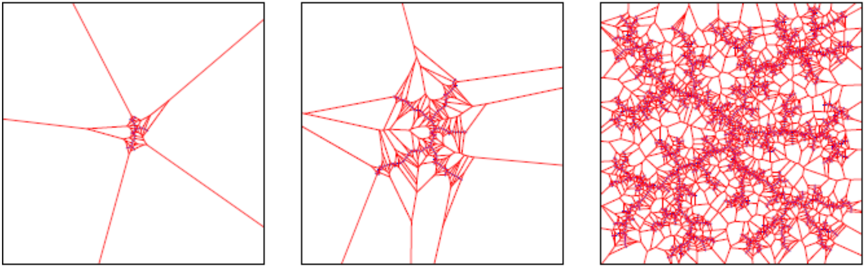
- What about the goal?
  - occasionally (e.g., every 100 iterations) choose the goal $\mathbf{x}_\tau$ as a sample and check if it can be connected to the tree
- Need distance function to find the nearest configurations in C (e.g., distance along the surface of a torus for a 2 link manipulator)
- A controller to track a line in C-space might be hard to design. We do not have to connect the configurations all the way. Instead, a local steering function with small step size ϵ can be used to get closer to the second configuration.
- To avoid constructing the configuration obstacle space C_{obs} explicitly, we need to do collision checking for the robot body in the workspace

Sampling in RRTs

- The vanilla RRT algorithm provides uniform coverage of space

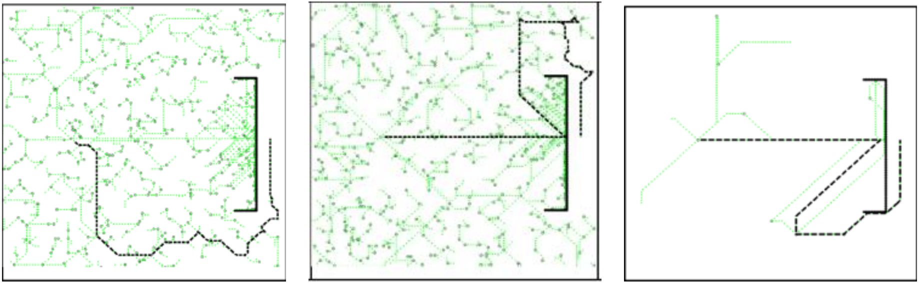


- Alternatively, the growth may be biased by the largest Voronoi region



Sampling in RRTs

- Goal-biased sampling: with probability $(1 - p_g)$, $\mathbf{x}_{rand}$ is chosen as a uniform sample in C_{free} and with probability p_g, $\mathbf{x}_{rand} = \mathbf{x}_T$

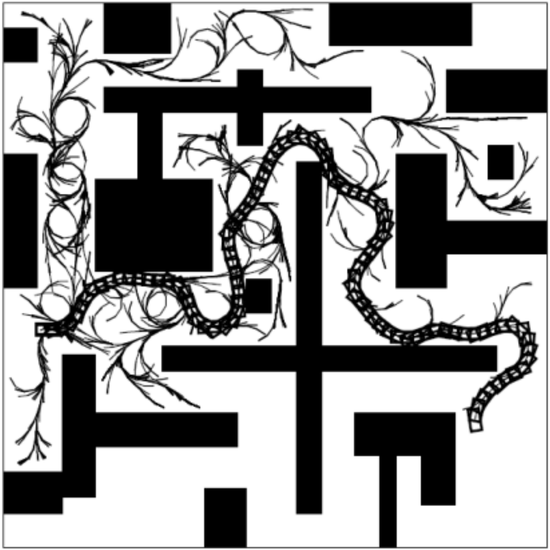


(a) $p_g = 0$ (b) $p_g = 0.1$ (c) $p_g = 0.5$

Handling Robot Dynamics with Steer$_\epsilon$()

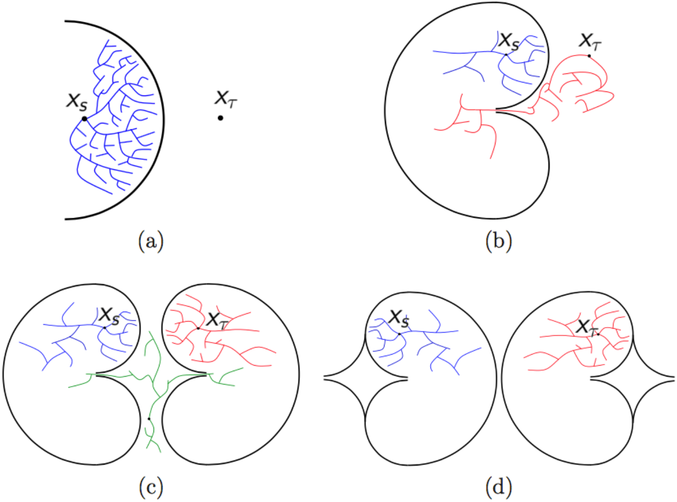
- $\textsc{Steer}_\epsilon()$ extends the tree towards a given random sample $\mathbf{x}_{rand}$
- Consider a car-like robot with non-holonomic constraints (no sideways motion) in $SE(2)$. Obtaining a feasible path from $\mathbf{x}_{rand} = (0, 0, 90^\circ)$ to $\mathbf{x}_{nearest} = (1, 0, 90^\circ)$ is as challenging as the original planning problem
- $\textsc{Steer}_\epsilon()$ resolves this by not requiring the motion to get all the way to $\mathbf{x}_{rand}$. Instead, apply the best control input for a fixed duration to obtain $\mathbf{x}_{new}$ and a dynamically feasible trajectory to it
- See: Y. Li, Z. Littlefield, K. Bekris, "Asymptotically optimal sampling-based kinodynamic planning," The International Journal of Robotics Research, 2016.

Example: 5 DOF Kinodynamic Planning for a Car



Bug Traps

- Growing two trees, one from start and one for goal, often has better performance in practice



Bi-Directional RRT

Algorithm Bi-Directional RRT

$V_a \leftarrow \{\mathbf{x}_s\};\ E_a \leftarrow \emptyset;\ V_b \leftarrow \{\mathbf{x}_\tau\};\ E_b \leftarrow \emptyset$
for $i = 1 \ldots n$ **do**
  $\mathbf{x}_{rand} \leftarrow \textsc{SampleFree}()$
  $\mathbf{x}_{nearest} \leftarrow \textsc{Nearest}((V_a, E_a), \mathbf{x}_{rand})$
  $\mathbf{x}_{new} \leftarrow \textsc{Steer}(\mathbf{x}_{nearest}, \mathbf{x}_{rand})$
  if $\mathbf{x}_{new} \neq \mathbf{x}_{nearest}$ **then**
    $V_a \leftarrow V_a \cup \{\mathbf{x}_{new}\};\ E_a \leftarrow \{(\mathbf{x}_{nearest}, \mathbf{x}_{new}), (\mathbf{x}_{new}, \mathbf{x}_{nearest})\}$
    $\mathbf{x}'_{nearest} \leftarrow \textsc{Nearest}((V_b, E_b), \mathbf{x}_{new})$
    $\mathbf{x}'_{new} \leftarrow \textsc{Steer}(\mathbf{x}'_{nearest}, \mathbf{x}_{new})$
    if $\mathbf{x}'_{new} \neq \mathbf{x}'_{nearest}$ **then**
      $V_b \leftarrow V_b \cup \{\mathbf{x}'_{new}\};\ E_b \leftarrow \{(\mathbf{x}'_{nearest}, \mathbf{x}'_{new}), (\mathbf{x}'_{new}, \mathbf{x}'_{nearest})\}$
    if $\mathbf{x}'_{new} = \mathbf{x}_{new}$ **then return** SOLUTION
  if $|V_b| < |V_a|$ **then** $\textsc{Swap}((V_a, E_a), (V_b, E_b))$
return FAILURE

RRT-Connect (J. Kuffner and S. LaValle, ICRA, 2000)

- Bi-directional tree + attempts to connect the two trees at every iteration

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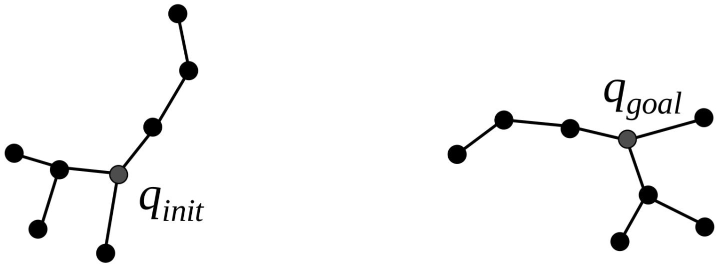
Algorithm RRT-Connect

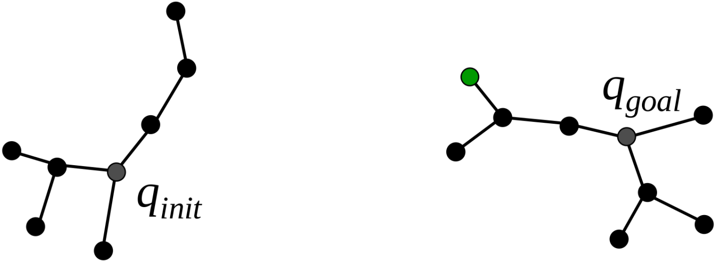
$V_a \leftarrow \{\mathbf{x}_s\}$; $E_a \leftarrow \emptyset$; $V_b \leftarrow \{\mathbf{x}_\tau\}$; $E_b \leftarrow \emptyset$
for $i = 1 \dots n$ **do**
    $\mathbf{x}_{rand} \leftarrow \textsc{SampleFree}()$
    if not $\textsc{Extend}((V_a, E_a), \mathbf{x}_{rand}) =$ Trapped **then**
        if $\textsc{Connect}((V_b, E_b), \mathbf{x}_{new}) =$ Reached **then**   ▷ $\mathbf{x}_{new}$ was just added to (V_a, E_a)
            return $\textsc{Path}((V_a, E_a), (V_b, E_b))$
    $\textsc{Swap}((V_a, E_a), (V_b, E_b))$
return Failure
function $\textsc{Extend}((V, E), \mathbf{x})$
    $\mathbf{x}_{nearest} \leftarrow \textsc{Nearest}((V, E), \mathbf{x})$
    $\mathbf{x}_{new} \leftarrow \textsc{Steer}_\epsilon(\mathbf{x}_{nearest}, \mathbf{x})$
    if $\textsc{CollisionFree}(\mathbf{x}_{nearest}, \mathbf{x}_{new})$ **then**
        $V \leftarrow \{\mathbf{x}_{new}\}$; $E \leftarrow \{(\mathbf{x}_{nearest}, \mathbf{x}_{new}), (\mathbf{x}_{new}, \mathbf{x}_{nearest})\}$
        if $\mathbf{x}_{new} = \mathbf{x}$ **then return** Reached **else return** Advanced
    return Trapped
function $\textsc{Connect}((V, E), \mathbf{x})$
    repeat $status \leftarrow \textsc{Extend}((V, E), \mathbf{x})$ **until** $status \neq Advanced$
    return $status$

Example: Single RRT-Connect Iteration



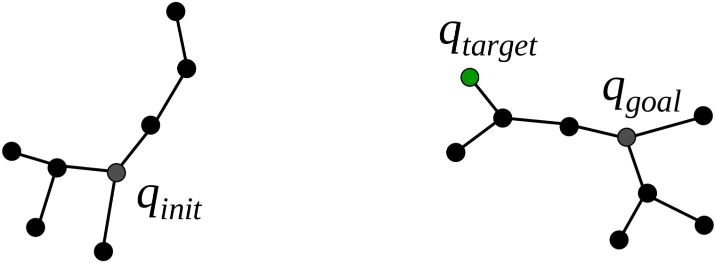
Example: Single RRT-Connect Iteration

- One tree is grown to a random target



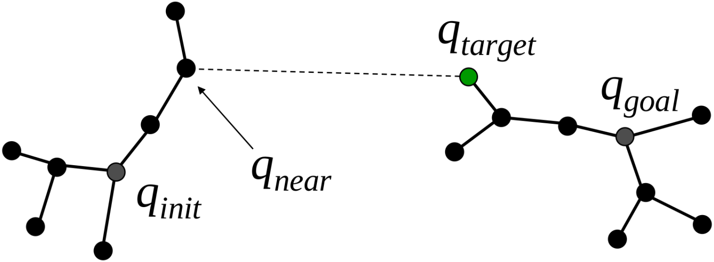
Example: Single RRT-Connect Iteration

- The new node becomes a target for the other tree



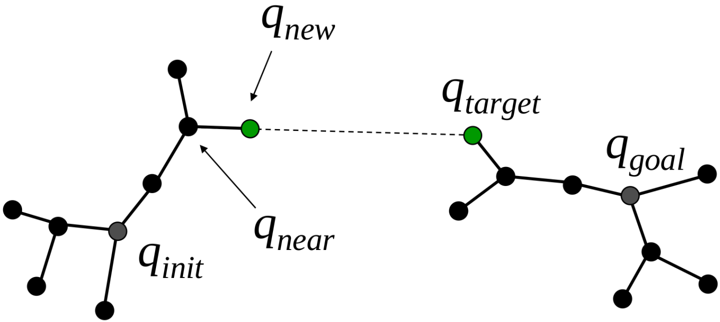
Example: Single RRT-Connect Iteration

- Determine the nearest node to the target



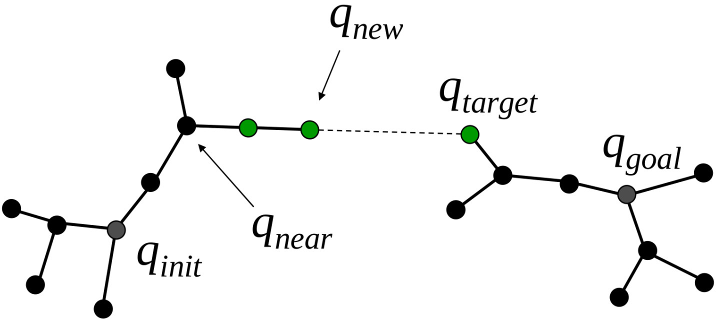
Example: Single RRT-Connect Iteration

- Try to add a new collision-free branch



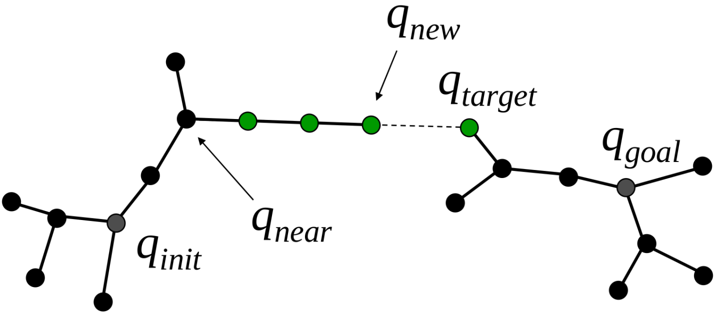
Example: Single RRT-Connect Iteration

- If successful, keep extending the branch



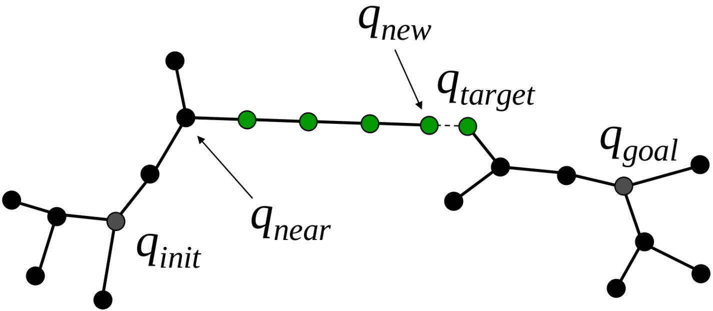
Example: Single RRT-Connect Iteration

- If successful, keep extending the branch



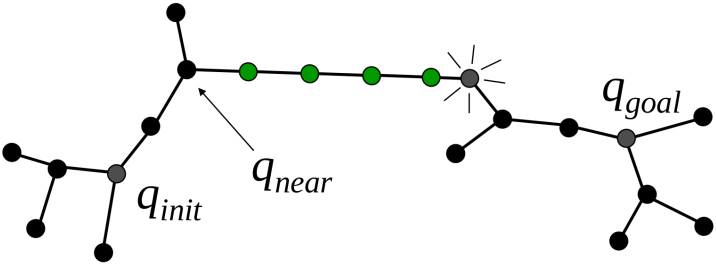
Example: Single RRT-Connect Iteration

- If successful, keep extending the branch



Example: Single RRT-Connect Iteration

- If the branch reaches all the way to the target, a feasible path is found!



Example: Single RRT-Connect Iteration

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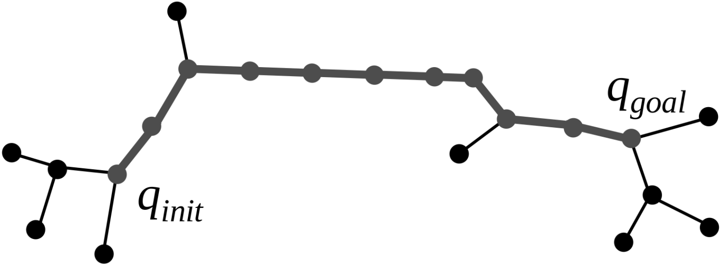
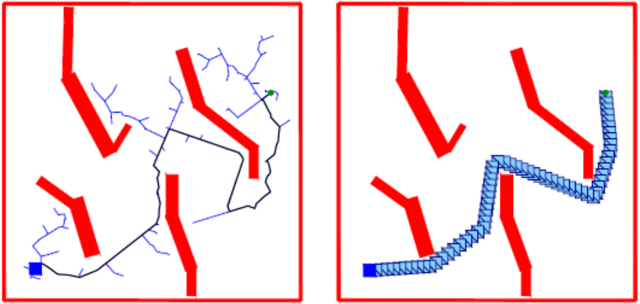
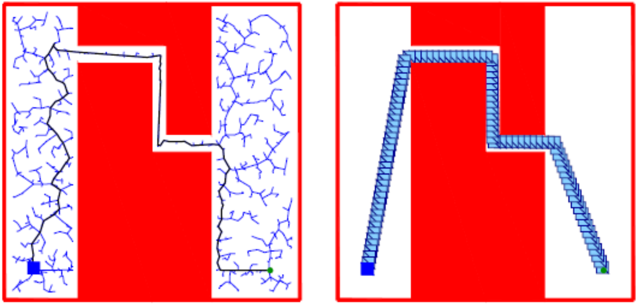


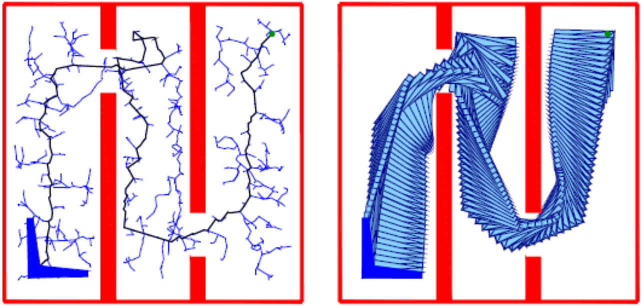
Example: RRT-Connect



Example: RRT-Connect



Example: RRT-Connect

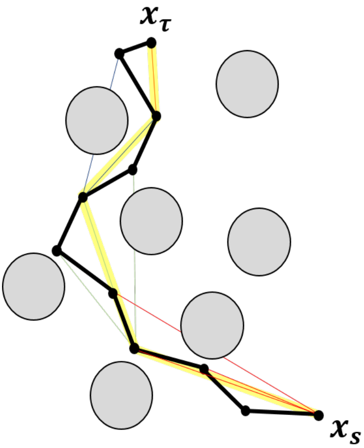


Why Are RRTs So Popular?

- The algorithm is very simple once the main subroutines are implemented:
  - Random sample generator
  - Nearest neighbor search
  - Collision checker
  - Steer function
- Pros:
  - A sparse graph requires little memory and computation
  - RRTs find feasible paths quickly in practice
  - Can add heuristic function, e.g., bias the sampling towards the goal (see Gammell et al., BIT*, IJRR, 2020)
- Cons:
  - Paths may be suboptimal and require smoothing as a post-processing step
  - Finding a path in highly constrained environments (e.g., maze) is challenging

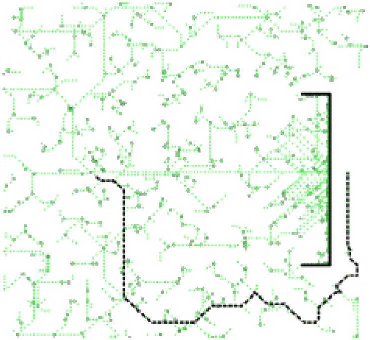
Path Smoothing

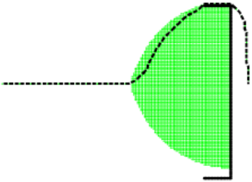
- Start with $\mathbf{x}_1 = \mathbf{x}_s$
- Make connections to subsequent points on the path $\mathbf{x}_2, \mathbf{x}_3, \mathbf{x}_4, \cdots$
- When a connection collides with obstacles, add the previous point to the smoothed path
- Continue smoothing from this point on



Search-Based vs Sampling-Based Planning

- RRT:
  - A sparse graph requires little memory and computation
  - Computed paths may be suboptimal and require smoothing
- Weighted A*:
  - Systematic exploration may require a lot of memory and computation
  - Returns a path with (sub)optimality guarantees





Outline

Search-Based vs Sampling-Based Planning

Probabilistic Roadmap

Rapidly Exploring Random Tree

RRT*

RRT: Probabilistic Completeness but No Optimality

- RRT and RRT-Connect are **probabilistically complete**: the probability that a feasible path will be found, if one exists, approaches 1 exponentially as the number of samples approaches infinity

- Assuming C_{free} is connected, bounded, and open, for any $\mathbf{x} \in \mathcal{C}_{free}$, $\lim_{N\to\infty} \mathbb{P}(\|\mathbf{x}-\mathbf{x}_{nearest}\| < \epsilon) = 1$, where $\mathbf{x}_{nearest}$ is the closest node to $\mathbf{x}$ in G

- RRT is **not optimal**: the probability that RRT converges to an optimal solution, as the number of samples approaches infinity, is zero under reasonable technical assumptions (S. Karaman, E. Frazzoli, RSS'10)

- **Problem with RRT**: once we build a tree, we never modify it

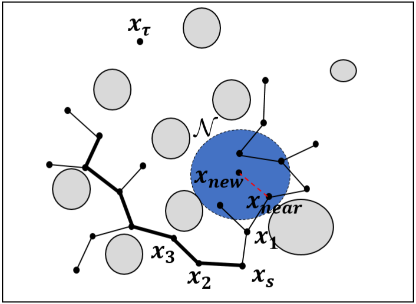
- **RRT*** (S. Karaman and E. Frazzoli, "Incremental Sampling-based Algorithms for Optimal Motion Planning," IJRR, 2010)
  - RRT + rewiring of the tree to ensure asymptotic optimality
  - Contains two steps: **extend** (similar to RRT) and **rewire** (new)

RRT*: Extend Step

- Generate a new potential node $\mathbf{x}_{new}$ identically to RRT
- Instead of finding the closest node in the tree, find all nodes within a neighborhood $\mathcal{N}$ of radius $\min\{r^*, \epsilon\}$ where

$$r^* > 2\left(1+\frac{1}{d}\right)^{1/d}\left(\frac{Vol(C_{free})}{Vol(\text{Unit d-ball})}\right)^{1/d}\left(\frac{\log|V|}{|V|}\right)^{(1/d)}$$

- Let $\mathbf{x}_{nearest} = \arg\min_{\mathbf{x}_{near}\in\mathcal{N}} g(\mathbf{x}_{near}) + c(\mathbf{x}_{near}, \mathbf{x}_{new})$ be the node in $\mathcal{N}$ on the currently known shortest path from $\mathbf{x}_s$ to $\mathbf{x}_{new}$

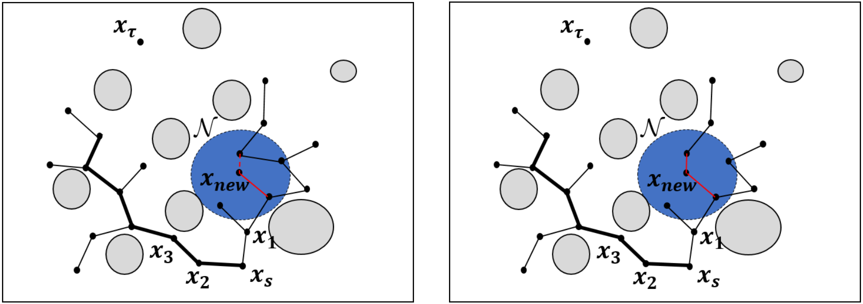


- $V \leftarrow V \cup \{\mathbf{x}_{new}\}$
- $E \leftarrow E \cup \{(\mathbf{x}_{nearest}, \mathbf{x}_{new})\}$
- Set the label of $\mathbf{x}_{new}$ to:

$$g(\mathbf{x}_{new}) = g(\mathbf{x}_{nearest}) + c(\mathbf{x}_{nearest}, \mathbf{x}_{new})$$

RRT*: Rewire Step

- Check all nodes $\mathbf{x}_{near} \in \mathcal{N}$ to see if re-routing through $\mathbf{x}_{new}$ reduces the path length (**label correcting!**)
- If $g(\mathbf{x}_{new}) + c(\mathbf{x}_{new}, \mathbf{x}_{near}) < g(\mathbf{x}_{near})$, then remove the edge between $\mathbf{x}_{near}$ and its parent and add a new edge between $\mathbf{x}_{near}$ and $\mathbf{x}_{new}$

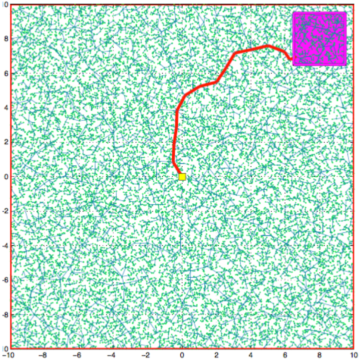


RRT*

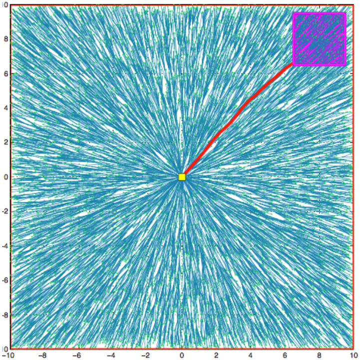
Algorithm RRT*

$V \leftarrow \{\mathbf{x}_s\}$; $E \leftarrow \emptyset$
for $i = 1 \dots n$ **do**
  $\mathbf{x}_{rand} \leftarrow \textsc{SampleFree}()$
  $\mathbf{x}_{nearest} \leftarrow \textsc{Nearest}((V, E), \mathbf{x}_{rand})$
  $\mathbf{x}_{new} \leftarrow \textsc{Steer}(\mathbf{x}_{nearest}, \mathbf{x}_{rand})$
  if $\textsc{CollisionFree}(\mathbf{x}_{nearest}, \mathbf{x}_{new})$ **then**
    $X_{near} \leftarrow \textsc{Near}((V, E), \mathbf{x}_{new}, \min\{r^*, \epsilon\})$
    $V \leftarrow V \cup \{\mathbf{x}_{new}\}$
    $c_{min} \leftarrow \textsc{Cost}(\mathbf{x}_{nearest}) + \textsc{Cost}(\mathit{Line}(\mathbf{x}_{nearest}, \mathbf{x}_{new}))$
    for $\mathbf{x}_{near} \in X_{near}$ **do** ▷ Extend along a minimum-cost path
      if $\textsc{CollisionFree}(\mathbf{x}_{near}, \mathbf{x}_{new})$ **then**
        if $\textsc{Cost}(\mathbf{x}_{near}) + \textsc{Cost}(\mathit{Line}(\mathbf{x}_{near}, \mathbf{x}_{new})) < c_{min}$ **then**
          $\mathbf{x}_{min} \leftarrow \mathbf{x}_{near}$
          $c_{min} \leftarrow \textsc{Cost}(\mathbf{x}_{near}) + \textsc{Cost}(\mathit{Line}(\mathbf{x}_{near}, \mathbf{x}_{new}))$
    $E \leftarrow E \cup \{(\mathbf{x}_{min}, \mathbf{x}_{new})\}$
    for $\mathbf{x}_{near} \in X_{near}$ **do** ▷ Rewire the tree
      if $\textsc{CollisionFree}(\mathbf{x}_{new}, \mathbf{x}_{near})$ **then**
        if $\textsc{Cost}(\mathbf{x}_{new}) + \textsc{Cost}(\mathit{Line}(\mathbf{x}_{new}, \mathbf{x}_{near})) < \textsc{Cost}(\mathbf{x}_{near})$ **then**
          $\mathbf{x}_{parent} \leftarrow \textsc{Parent}(\mathbf{x}_{near})$
          $E \leftarrow (E \setminus \{(\mathbf{x}_{parent}, \mathbf{x}_{near})\}) \cup \{(\mathbf{x}_{new}, \mathbf{x}_{near})\}$
return $G = (V, E)$

RRT vs RRT*



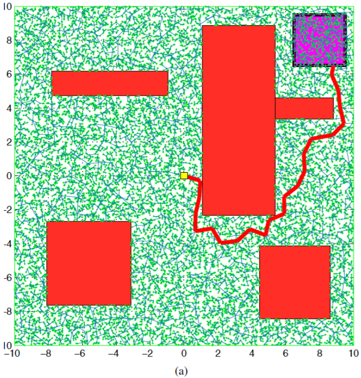
(a) RRT



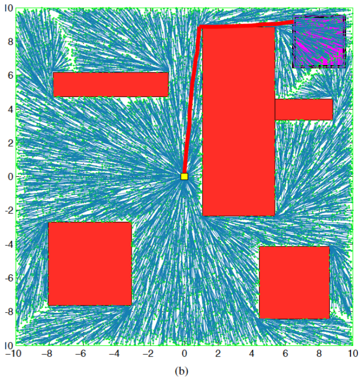
(b) RRT*

- Same nodes in the tree, only the edge connections are different. Notice how RRT* edges are almost straight lines (optimal paths).

RRT vs RRT*



(a) RRT



(b) RRT*

- Same nodes in the tree, only the edge connections are different. Notice how RRT* edges are almost straight lines (optimal paths).

Sampling-Based Planning in Practice

- A. Orthey, C. Chamzas, L. Kavraki, "Sampling-Based Motion Planning: A Comparative Review," Annual Review of Control, Robotics, and Autonomous Systems, 2024

  https://doi.org/10.1146/annurev-control-061623-094742

- The Open Motion Planning Library (OMPL):

  https://ompl.kavrakilab.org/

- Motion Planning Templates (MPT):

  https://robotics.cs.unc.edu/mpt/