

ECE276B: Planning & Learning in Robotics

Lecture 8: Sampling-based Planning

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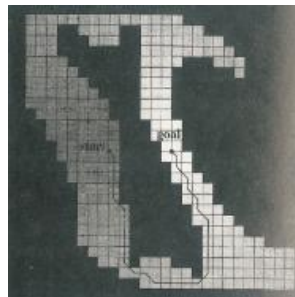
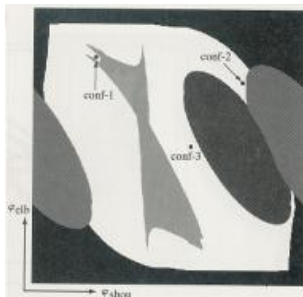
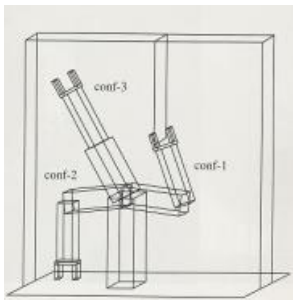
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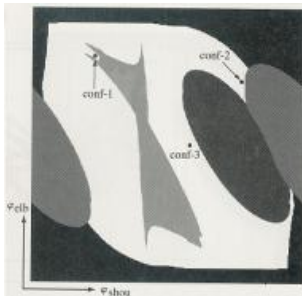
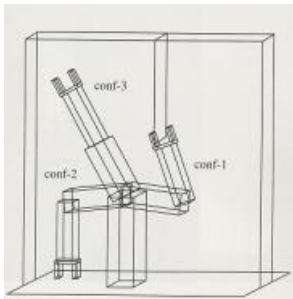
Search-based vs Sampling-based Planning

- ▶ Search-based planning:
 - ▶ Generates a systematic discrete representation (graph) of C_{free}
 - ▶ Searches the representation for a path guaranteeing to find one if it exists (resolution complete)
 - ▶ Can interleave the graph construction with the search, i.e., nodes added only when necessary
 - ▶ Provides finite-tiem suboptimality bounds on the solution
 - ▶ Computationally expensive in high dimensions



Search-based vs. Sampling-based Planning

- ▶ Sampling-based planning:
 - ▶ Generates a sparse sample-based graph of C_{free}
 - ▶ Searches the graph for a path guaranteeing that the probability of finding one if it exists approaches 1 as the number of iterations $\rightarrow \infty$ (probabilistically complete)
 - ▶ Can interleave the graph construction with the search, i.e., samples added only when necessary
 - ▶ Provides asymptotic suboptimality bounds on the solution
 - ▶ Well-suited for high-dimensional planning: faster and requires less memory than search-based planning in many domains



Motion Planning Problem

- ▶ Configuration space: C ; Obstacle space: C_{obs} ; Free space: C_{free}
- ▶ Initial state: $\mathbf{x}_s \in C_{free}$; Goal state: $\mathbf{x}_\tau \in C_{free}$
- ▶ **Path**: a continuous function $\rho : [0, 1] \rightarrow C$; Set of all paths: \mathcal{P}
- ▶ **Feasible path**: a continuous function $\rho : [0, 1] \rightarrow C_{free}$ such that $\rho(0) = \mathbf{x}_s$ and $\rho(1) = \mathbf{x}_\tau$; Set of all feasible paths: $\mathcal{P}_{s,\tau}$
- ▶ **Motion Planning Problem** Given a path planning problem $(C_{free}, \mathbf{x}_s, \mathbf{x}_\tau)$ and a cost function $J : \mathcal{P} \rightarrow \mathbb{R}_{\geq 0}$, find a feasible path ρ^* such that:

$$J(\rho^*) = \min_{\rho \in \mathcal{P}_{s,\tau}} J(\rho)$$

Report failure if no such path exists.

Primitive Procedures for Sampling-based Motion Planning

- ▶ **SAMPLE**: returns iid samples from C
- ▶ **SAMPLEFREE**: returns iid samples from C_{free}
- ▶ **NEAREST**: given a graph $G = (V, E)$ with $V \subset C$ and a point $\mathbf{x} \in C$, returns a vertex $\mathbf{v} \in V$ that is closest to \mathbf{x} :

$$\text{NEAREST}((V, E), \mathbf{x}) := \arg \min_{\mathbf{v} \in V} \|\mathbf{x} - \mathbf{v}\|$$

- ▶ **NEAR**: given a graph $G = (V, E)$ with $V \subset C$, a point $\mathbf{x} \in C$, and $r > 0$, returns the vertices in V that are within a distance r from \mathbf{x} :

$$\text{NEAR}((V, E), \mathbf{x}, r) := \{\mathbf{v} \in V \mid \|\mathbf{x} - \mathbf{v}\| \leq r\}$$

- ▶ **STEER**: given points $\mathbf{x}, \mathbf{y} \in C$ and $\epsilon > 0$, returns a point $\mathbf{z} \in C$ that minimizes $\|\mathbf{z} - \mathbf{y}\|$ while remaining within ϵ from \mathbf{x} :

$$\text{STEER}_\epsilon(\mathbf{x}, \mathbf{y}) := \arg \min_{\mathbf{z}: \|\mathbf{z} - \mathbf{x}\| \leq \epsilon} \|\mathbf{z} - \mathbf{y}\|$$

- ▶ **COLLISIONFREE**: given points $\mathbf{x}, \mathbf{y} \in C$, returns **TRUE** if the line segment between \mathbf{x} and \mathbf{y} lies in C_{free} and **FALSE** otherwise.

Probabilistic Roadmap (PRM)

Step 1. Construction Phase: Build a roadmap (graph) G which, hopefully, should be accessible from any point in C_{free}

- ▶ **Nodes:** randomly sampled valid configurations $\mathbf{x}_i \in C_{free}$
- ▶ **Edges:** added between samples that are easy to connect with a simple local controller (e.g., follow straight line)

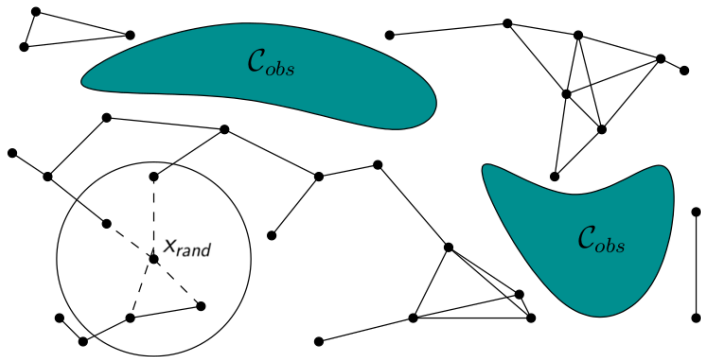


Step 2. Query Phase: Given a start configuration \mathbf{x}_s and goal configuration \mathbf{x}_T , connect them to the roadmap G , then search the augmented roadmap for a shortest path from \mathbf{x}_s to \mathbf{x}_T

▶ **Pros and Cons:**

- ▶ Simple and highly effective in high dimensions
- ▶ Can result in suboptimal paths, no guarantees on suboptimality
- ▶ Difficulty with narrow passages
- ▶ Useful for multiple queries with different start and goal in the same environment

Step 1: Construction Phase



Step 1: Construction Phase

Algorithm 1 PRM (construction phase)

```
1:  $V \leftarrow \emptyset; E \leftarrow \emptyset$ 
2: for  $i = 1, \dots, n$  do
3:    $\mathbf{x}_{rand} \leftarrow \text{SAMPLEFREE}()$ 
4:    $V \leftarrow V \cup \{\mathbf{x}_{rand}\}$ 
5:   for  $\mathbf{x} \in \text{NEAR}((V, E), \mathbf{x}_{rand}, r)$  do ▷ May use  $k$  nearest vertices
6:     if (not  $G.\text{same\_component}(\mathbf{x}_{rand}, \mathbf{x})$ ) and  $\text{COLLISIONFREE}(\mathbf{x}_{rand}, \mathbf{x})$  then
7:        $E \leftarrow E \cup \{(\mathbf{x}_{rand}, \mathbf{x}), (\mathbf{x}, \mathbf{x}_{rand})\}$ 
8: return  $G = (V, E)$ 
```

▶ $G.\text{same_component}(\mathbf{x}_{rand}, \mathbf{x})$

- ▶ ensures that \mathbf{x} and \mathbf{x}_{rand} are in different components of G
- ▶ every connection decreases the number of connected components in G
- ▶ efficient implementation using union-find algorithms
- ▶ may be replaced by $G.\text{vertex_degree}(\mathbf{x}) < K$ for some fixed K (e.g., $K = 15$) if it is important to generate multiple alternative paths

Asymptotically Optimal Probabilistic Roadmap

- ▶ S. Karaman and E. Frazzoli, “Incremental Sampling-based Algorithms for Optimal Motion Planning,” IJRR, 2010.
- ▶ To achieve an asymptotically optimal PRM, the connection radius r should decrease such that the average number of connections attempted from a roadmap vertex is proportional to $\log(n)$:

$$r^* > 2 \left(1 + \frac{1}{d}\right)^{1/d} \left(\frac{\text{Vol}(C_{\text{free}})}{\text{Vol}(\text{Unit } d\text{-ball})}\right)^{1/d} \left(\frac{\log(n)}{n}\right)^{1/d}$$

Algorithm 2 PRM*

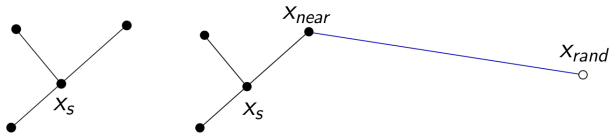
```
1:  $V \leftarrow \{\mathbf{x}_s\} \cup \{\text{SAMPLEFREE}()\}_{i=1}^n$ ;  $E \leftarrow \emptyset$ 
2: for  $\mathbf{v} \in V$  do
3:   for  $\mathbf{x} \in \text{NEAR}((V, E), \mathbf{v}, r^*) \setminus \{\mathbf{v}\}$  do
4:     if  $\text{COLLISIONFREE}(\mathbf{v}, \mathbf{x})$  then
5:        $E \leftarrow E \cup \{(\mathbf{v}, \mathbf{x}), (\mathbf{x}, \mathbf{v})\}$ 
6: return  $G = (V, E)$ 
```

PRM vs RRT

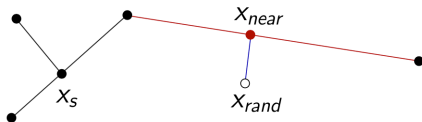
- ▶ **PRM**: a graph constructed from random samples. It can be searched for a path whenever a start node \mathbf{x}_s and goal node \mathbf{x}_t are specified. PRMs are well-suited for repeated planning between different pairs of \mathbf{x}_s and \mathbf{x}_t (*multiple queries*)
- ▶ **RRT**: a tree constructed from random samples with root \mathbf{x}_s . The tree is grown until it contains a path to \mathbf{x}_t . RRTs are well-suited for single-shot planning between a single pair of \mathbf{x}_s and \mathbf{x}_t (*single query*)
- ▶ **Rapidly Exploring Random Tree (RRT)**:
 - ▶ One of the most popular planning techniques
 - ▶ Introduced by Steven LaValle in 1998
 - ▶ Many, many, many extensions and variants (articulated robots, kinematics, dynamics, differential constraints)
 - ▶ There exist extensions of RRTs that try to reuse a previously constructed tree when replanning in response to map updates

Rapidly Exploring Random Tree (RRT)

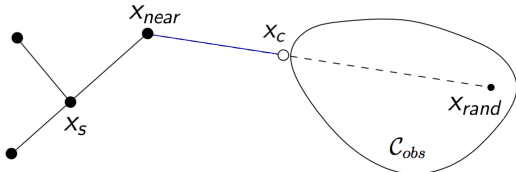
- ▶ Sample a new configuration x_{rand} , find the nearest neighbor $x_{nearest}$ in G and connect them:



- ▶ (Optional) if $x_{nearest}$ lies on an existing edge, then split the edge:



- ▶ If there is an obstacle, the edge travels up to the obstacle boundary, as far as allowed by a collision detection algorithm



Rapidly Exploring Random Tree (RRT)

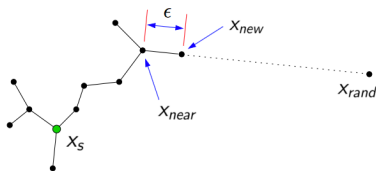
- ▶ What about the goal? Occasionally (e.g., every 100 iterations) add the goal configuration \mathbf{x}_T and see if it gets connected to the tree
- ▶ RRT can be implemented in the original workspace (need to do collision checking) or in configuration space
- ▶ Challenges with a C-Space implementation:
 - ▶ What distance function do we use to find the nearest configuration?
 - ▶ e.g., distance along the surface of a torus for a 2 link manipulator
 - ▶ An edge represents a path in C-Space. How do we construct a collision-free path between two configurations?
 - ▶ We do not have to connect the configurations all the way. Instead, use a small step size ϵ and a local steering function to get closer to the second configuration.

Rapidly Exploring Random Tree (RRT)

- ▶ **No preprocessing:** starting with an initial configuration \mathbf{x}_s build a graph (actually, tree) until the goal configuration \mathbf{x}_t is part of it

Algorithm 3 RRT

- 1: $V \leftarrow \{\mathbf{x}_s\}; E \leftarrow \emptyset$
 - 2: **for** $i = 1 \dots n$ **do**
 - 3: $\mathbf{x}_{rand} \leftarrow \text{SAMPLEFREE}()$
 - 4: $\mathbf{x}_{nearest} \leftarrow \text{NEAREST}((V, E), \mathbf{x}_{rand})$
 - 5: $\mathbf{x}_{new} \leftarrow \text{STEER}(\mathbf{x}_{nearest}, \mathbf{x}_{rand})$
 - 6: **if** $\text{COLLISIONFREE}(\mathbf{x}_{nearest}, \mathbf{x}_{new})$ **then**
 - 7: $V \leftarrow V \cup \{\mathbf{x}_{new}\}; E \leftarrow E \cup \{(\mathbf{x}_{nearest}, \mathbf{x}_{new})\}$
 - 8: **return** $G = (V, E)$
-

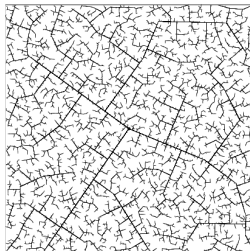


Rapidly Exploring Random Tree (RRT)

- ▶ RRT without ϵ (called Rapidly Exploring Dense Tree (RDT)):

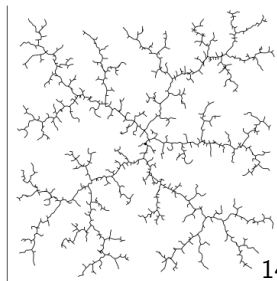
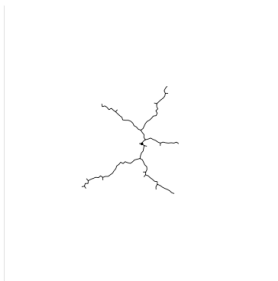
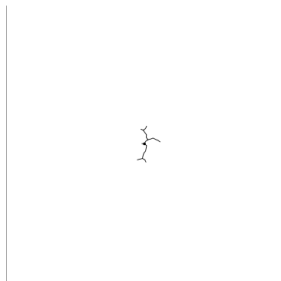


45 iterations



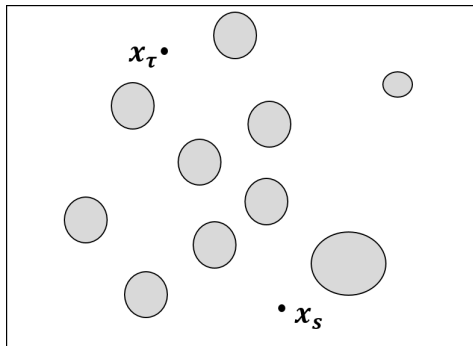
2345 iterations

- ▶ RRT with ϵ



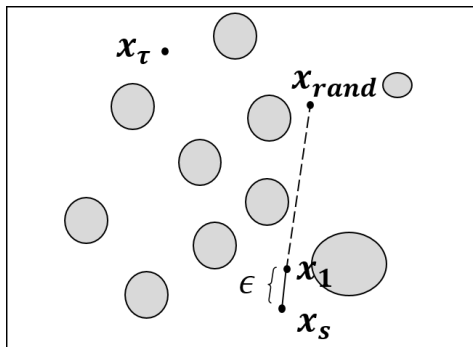
Example: RRT Algorithm

- ▶ Start node x_s
- ▶ Goal node x_t
- ▶ Gray obstacles



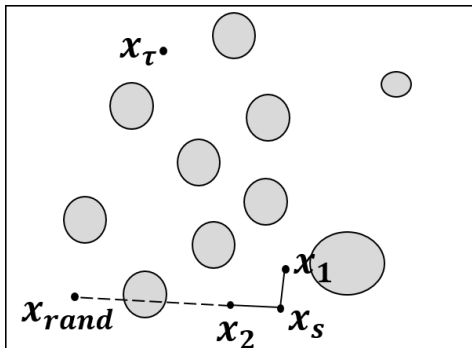
Example: RRT Algorithm

- ▶ Sample \mathbf{x}_{rand} in the workspace
- ▶ Steer from \mathbf{x}_s towards \mathbf{x}_{rand} by a fixed distance ϵ to get \mathbf{x}_1
- ▶ If the segment from \mathbf{x}_s to \mathbf{x}_1 is collision-free, insert \mathbf{x}_1 into the tree



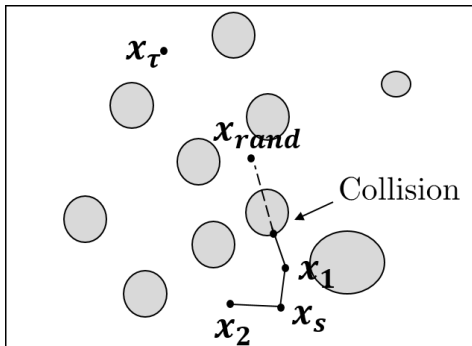
Example: RRT Algorithm

- ▶ Sample \mathbf{x}_{rand} in the workspace
- ▶ Find the closest node $\mathbf{x}_{nearest}$ to \mathbf{x}_{rand}
- ▶ Steer from $\mathbf{x}_{nearest}$ towards \mathbf{x}_{rand} by a fixed distance ϵ to get \mathbf{x}_2
- ▶ If the segment from $\mathbf{x}_{nearest}$ to \mathbf{x}_2 is collision-free, insert \mathbf{x}_2 into the tree



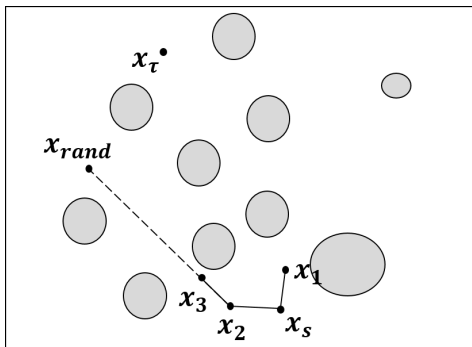
Example: RRT Algorithm

- ▶ Sample \mathbf{x}_{rand} in the workspace
- ▶ Find the closest node $\mathbf{x}_{nearest}$ to \mathbf{x}_{rand}
- ▶ Steer from $\mathbf{x}_{nearest}$ towards \mathbf{x}_{rand} by a fixed distance ϵ to get \mathbf{x}_3
- ▶ If the segment from $\mathbf{x}_{nearest}$ to \mathbf{x}_3 is collision-free, insert \mathbf{x}_3 into the tree



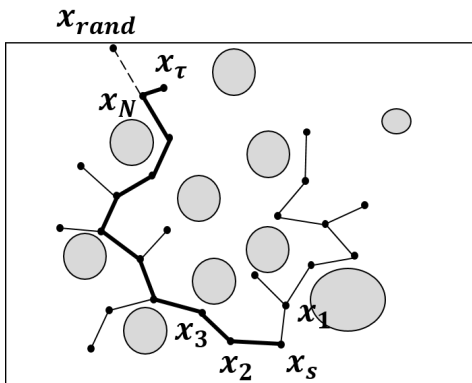
Example: RRT Algorithm

- ▶ Sample \mathbf{x}_{rand} in the workspace
- ▶ Find the closest node $\mathbf{x}_{nearest}$ to \mathbf{x}_{rand}
- ▶ Steer from $\mathbf{x}_{nearest}$ towards \mathbf{x}_{rand} by a fixed distance ϵ to get \mathbf{x}_3
- ▶ If the segment from $\mathbf{x}_{nearest}$ to \mathbf{x}_3 is collision-free, insert \mathbf{x}_3 into the tree



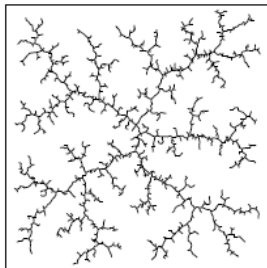
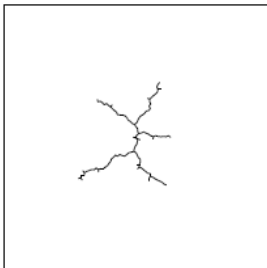
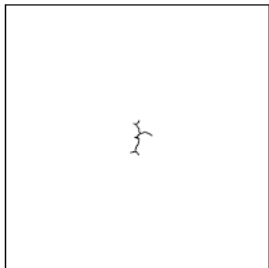
Example: RRT Algorithm

- ▶ Continue until a node that is a distance ϵ from the goal is generated
- ▶ Either terminate the algorithm or search for additional feasible paths

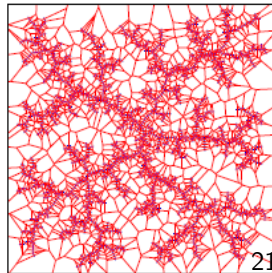
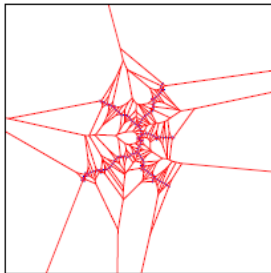
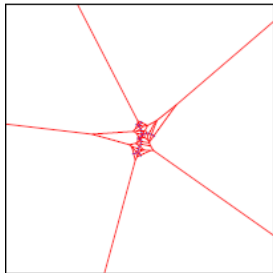


Sampling in RRTs

- ▶ The vanilla RRT algorithm provides uniform coverage of space

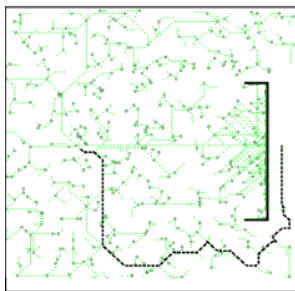


- ▶ Alternatively, the growth may be biased by the largest Voronoi region

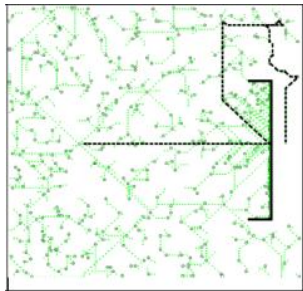


Sampling in RRTs

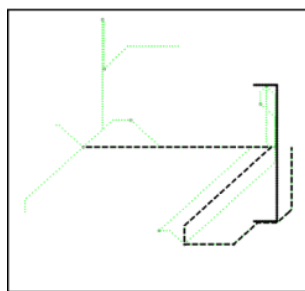
- ▶ Goal-biased sampling: with probability $(1 - p_g)$, \mathbf{x}_{rand} is chosen as a uniform sample in C_{free} and with probability p_g , $\mathbf{x}_{rand} = \mathbf{x}_\tau$



(a) $p_g = 0$



(b) $p_g = 0.1$

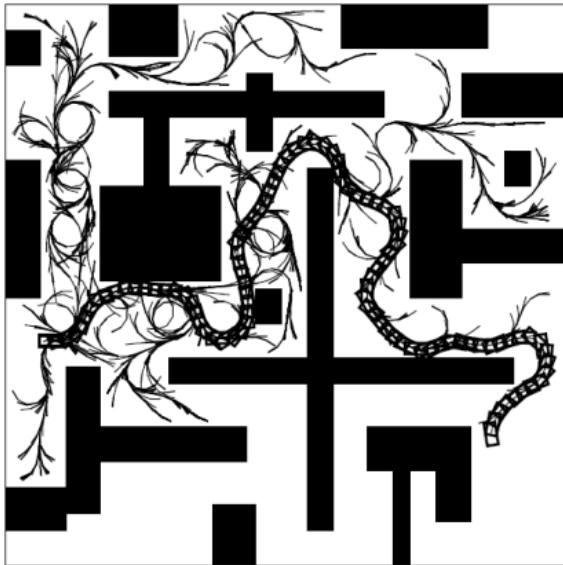


(c) $p_g = 0.5$

Handling Robot Dynamics with Steer()

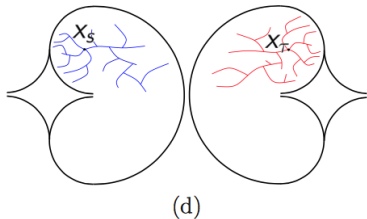
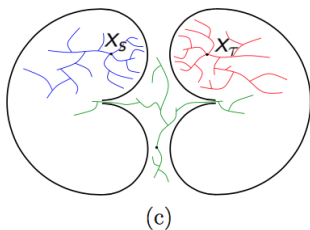
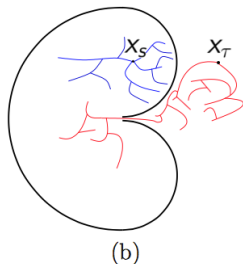
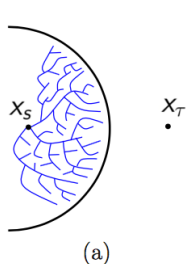
- ▶ Steer() extends the tree towards a given random sample \mathbf{x}_{rand}
- ▶ Consider a car-like robot with non-holonomic constraints (can't slide sideways) in $SE(2)$. Obtaining a feasible path from $\mathbf{x}_{rand} = (0, 0, 90^\circ)$ to $\mathbf{x}_{nearest} = (1, 0, 90^\circ)$ is as hard as the original problem
- ▶ Steer() resolves this by not requiring the motion to get all the way to \mathbf{x}_{rand} . We just apply the best control input for a fixed duration to obtain \mathbf{x}_{new} and a dynamically feasible trajectory to it

Example: 5 DOF Kinodynamic Planning for a Car



Bug Traps

- ▶ Growing two trees, one from start and one for goal, often has better performance in practice.



Bi-directional RRT

Algorithm 4 Bi-directional RRT

```
1:  $V_a \leftarrow \{\mathbf{x}_s\}; E_a \leftarrow \emptyset; V_b \leftarrow \{\mathbf{x}_t\}; E_b \leftarrow \emptyset$ 
2: for  $i = 1 \dots n$  do
3:    $\mathbf{x}_{rand} \leftarrow \text{SAMPLEFREE}()$ 
4:    $\mathbf{x}_{nearest} \leftarrow \text{NEAREST}((V_a, E_a), \mathbf{x}_{rand})$ 
5:    $\mathbf{x}_{new} \leftarrow \text{STEER}(\mathbf{x}_{nearest}, \mathbf{x}_{rand})$ 
6:   if  $\mathbf{x}_{new} \neq \mathbf{x}_{nearest}$  then
7:      $V_a \leftarrow V_a \cup \{\mathbf{x}_{new}\}; E_a \leftarrow \{(\mathbf{x}_{nearest}, \mathbf{x}_{new}), (\mathbf{x}_{new}, \mathbf{x}_{nearest})\}$ 
8:      $\mathbf{x}'_{nearest} \leftarrow \text{NEAREST}((V_b, E_b), \mathbf{x}_{new})$ 
9:      $\mathbf{x}'_{new} \leftarrow \text{STEER}(\mathbf{x}'_{nearest}, \mathbf{x}_{new})$ 
10:    if  $\mathbf{x}'_{new} \neq \mathbf{x}'_{nearest}$  then
11:       $V_b \leftarrow V_b \cup \{\mathbf{x}'_{new}\}; E_b \leftarrow \{(\mathbf{x}'_{nearest}, \mathbf{x}'_{new}), (\mathbf{x}'_{new}, \mathbf{x}'_{nearest})\}$ 
12:      if  $\mathbf{x}'_{new} = \mathbf{x}_{new}$  then return SOLUTION
13:    if  $|V_b| < |V_a|$  then SWAP $((V_a, E_a), (V_b, E_b))$ 
14: return FAILURE
```

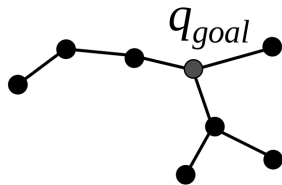
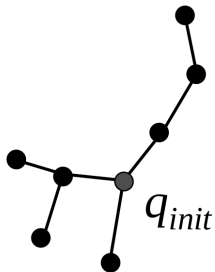
RRT-Connect (J. Kuffner and S. LaValle, ICRA, 2000)

- Bi-directional tree + attempts to connect the two trees at every iteration

Algorithm 5 RRT-Connect

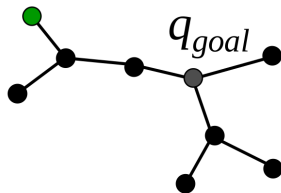
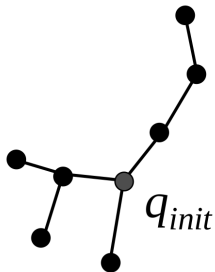
```
1:  $V_a \leftarrow \{\mathbf{x}_s\}; E_a \leftarrow \emptyset; V_b \leftarrow \{\mathbf{x}_t\}; E_b \leftarrow \emptyset$ 
2: for  $i = 1 \dots n$  do
3:    $\mathbf{x}_{rand} \leftarrow \text{SAMPLEFREE}()$ 
4:   if not  $\text{EXTEND}((V_a, E_a), \mathbf{x}_{rand}) = \text{Trapped}$  then
5:     if  $\text{CONNECT}((V_b, E_b), \mathbf{x}_{new}) = \text{Reached}$  then  $\triangleright \mathbf{x}_{new}$  was just added to  $(V_a, E_a)$ 
6:       return  $\text{PATH}((V_a, E_a), (V_b, E_b))$ 
7:    $\text{SWAP}((V_a, E_a), (V_b, E_b))$ 
8: return Failure
9: function  $\text{EXTEND}((V, E), \mathbf{x})$ 
10:   $\mathbf{x}_{nearest} \leftarrow \text{NEAREST}((V, E), \mathbf{x})$ 
11:   $\mathbf{x}_{new} \leftarrow \text{STEER}_\epsilon(\mathbf{x}_{nearest}, \mathbf{x})$ 
12:  if  $\text{COLLISIONFREE}(\mathbf{x}_{nearest}, \mathbf{x}_{new})$  then
13:     $V \leftarrow \{\mathbf{x}_{new}\}; E \leftarrow \{(\mathbf{x}_{nearest}, \mathbf{x}_{new}), (\mathbf{x}_{new}, \mathbf{x}_{nearest})\}$ 
14:    if  $\mathbf{x}_{new} = \mathbf{x}$  then return Reached else return Advanced
15:  return Trapped
16: function  $\text{CONNECT}((V, E), \mathbf{x})$ 
17:  repeat  $\text{status} \leftarrow \text{EXTEND}((V, E), \mathbf{x})$  until  $\text{status} \neq \text{Advanced}$ 
18:  return  $\text{status}$ 
```

Example: Single RRT-Connect Iteration



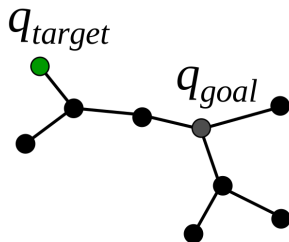
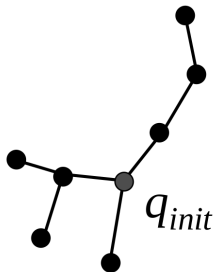
Example: Single RRT-Connect Iteration

- ▶ One tree is grown to a random target



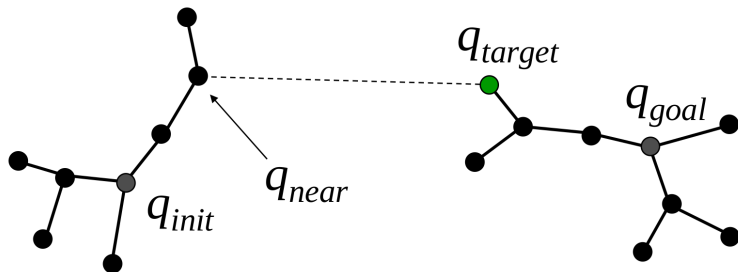
Example: Single RRT-Connect Iteration

- ▶ The new node becomes a target for the other tree



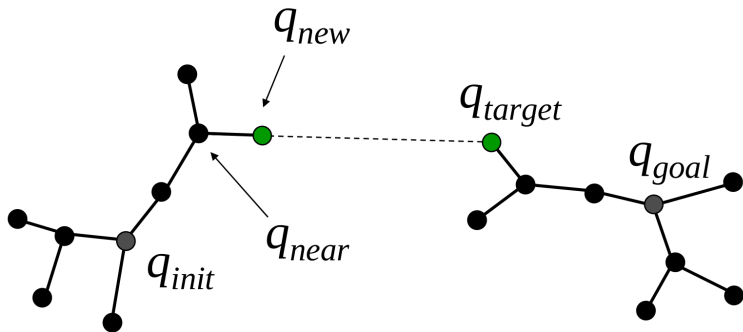
Example: Single RRT-Connect Iteration

- ▶ Determine the nearest node to the target



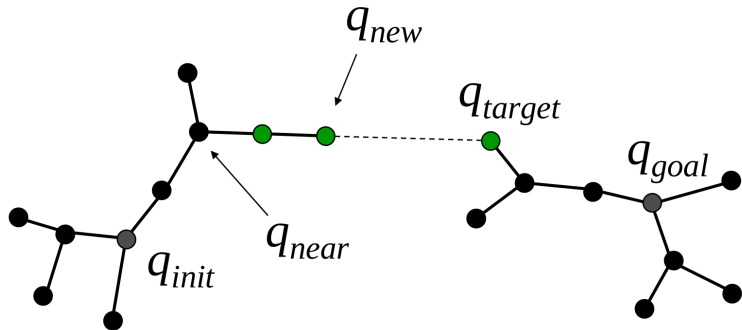
Example: Single RRT-Connect Iteration

- ▶ Try to add a new collision-free branch



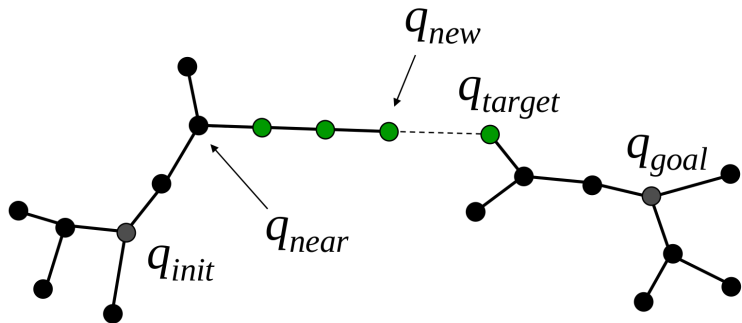
Example: Single RRT-Connect Iteration

- ▶ If successful, keep extending the branch



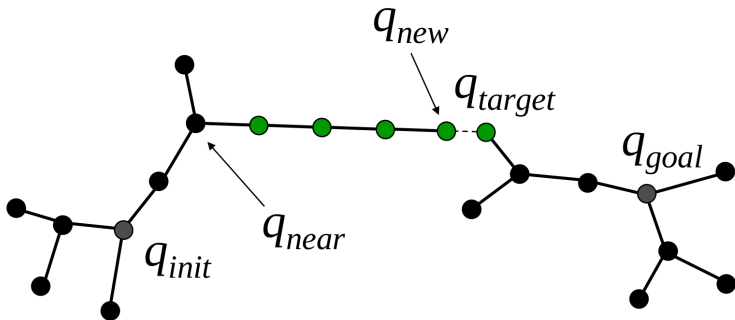
Example: Single RRT-Connect Iteration

- ▶ If successful, keep extending the branch



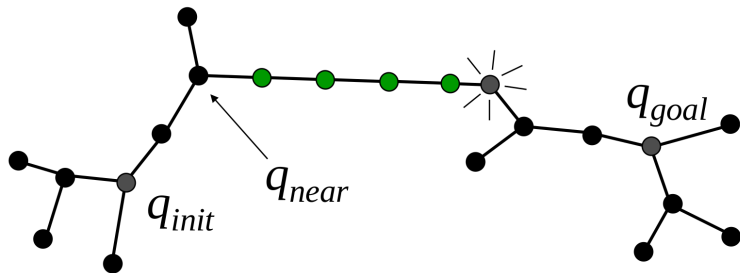
Example: Single RRT-Connect Iteration

- ▶ If successful, keep extending the branch



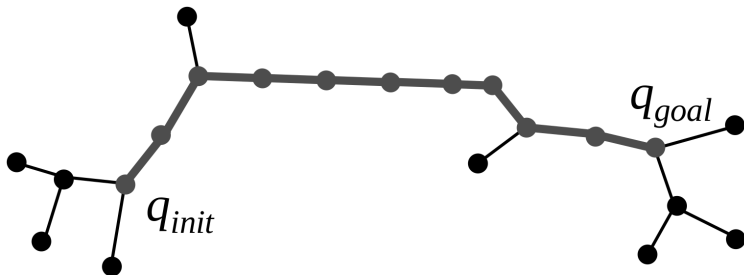
Example: Single RRT-Connect Iteration

- ▶ If the branch reaches all the way to the target, a feasible path is found!

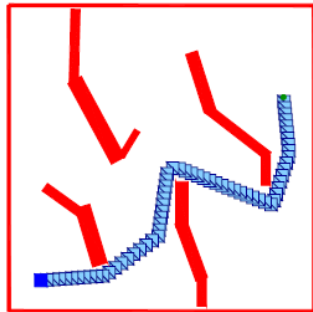


Example: Single RRT-Connect Iteration

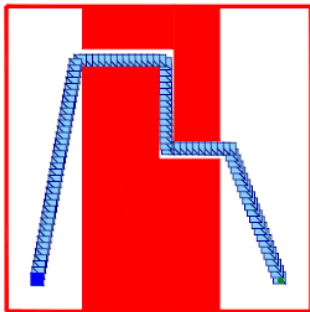
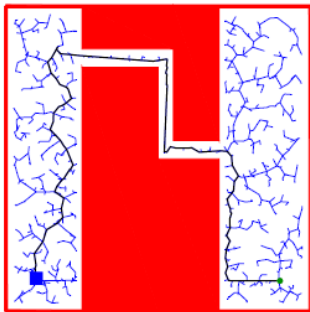
- ▶ If the branch reaches all the way to the target, a feasible path is found!



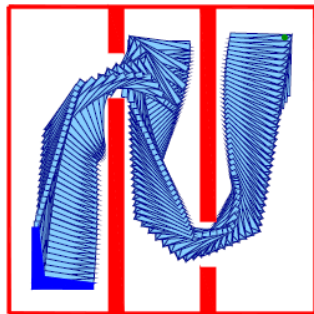
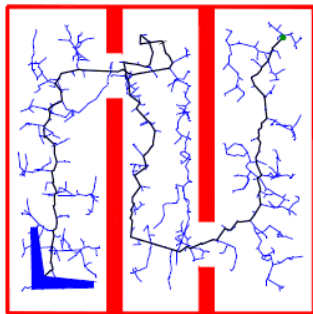
Example: RRT-Connect



Example: RRT-Connect



Example: RRT-Connect

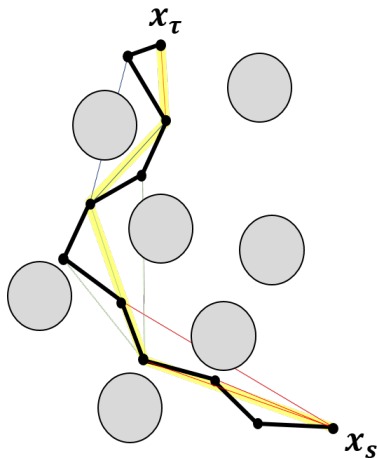


Why are RRTs so popular?

- ▶ The algorithm is very simple once the following subroutines are implemented:
 - ▶ Random sample generator
 - ▶ Nearest neighbor
 - ▶ Collision checker
 - ▶ Steer
- ▶ Pros:
 - ▶ A sparse graph requires little memory and computation
 - ▶ RRTs find feasible paths quickly in practice
 - ▶ Can add heuristics on top, e.g., bias the sampling towards the goal (see Gammell et al., BIT*, IJRR, 2020.)
- ▶ Cons:
 - ▶ Computed paths may be sub-optimal and require path smoothing as a post-processing step
 - ▶ Finding a feasible path in highly constrained environments (e.g., maze) is challenging

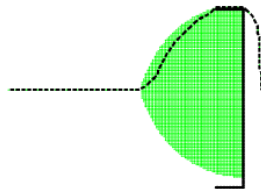
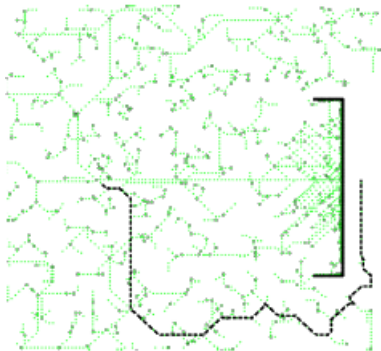
Path Smoothing

- ▶ Start with the initial point (1)
- ▶ Make connections to subsequent points in the path (2), (3), (4), ...
- ▶ When a connection collides with obstacles, add the previous waypoint to the smoothed path
- ▶ Continue smoothing from this point on



Search-based vs Sampling-based Planning

- ▶ RRT:
 - ▶ A sparse graph requires little memory and computation
 - ▶ Computed paths may be sub-optimal and require path smoothing
- ▶ Weighted A*:
 - ▶ Systematic exploration may require a lot of memory and computation
 - ▶ Returns a path with (sub-)optimality guarantees



RRT: Probabilistic Completeness but No Optimality

- ▶ RRT and RRT-Connect are **probabilistically complete**: the probability that a feasible path will be found if one exists, approaches 1 exponentially as the number of samples approaches infinity
- ▶ Assuming C_{free} is connected, bounded, and open, for any $x \in C_{free}$,
 $\lim_{N \rightarrow \infty} \mathbb{P}(\|\mathbf{x} - \mathbf{x}_{nearest}\| < \epsilon) = 1$, where $\mathbf{x}_{nearest}$ is the closest node to \mathbf{x} in G
- ▶ RRT is **not optimal**: the probability that RRT converges to an optimal solution, as the number of samples approaches infinity, is zero under reasonable technical assumptions (S. Karaman, E. Frazzoli, RSS'10)
- ▶ **Problem**: once we build an RRT we never modify it
- ▶ **RRT*** (S. Karaman and E. Frazzoli, "Incremental Sampling-based Algorithms for Optimal Motion Planning," IJRR, 2010)
 - ▶ RRT + rewiring of the tree to ensure asymptotic optimality
 - ▶ Contains two steps: **extend** (similar to RRT) and **rewire** (new)

RRT*: Extend Step

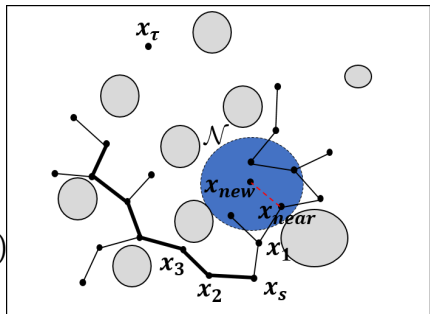
- ▶ Generate a new potential node \mathbf{x}_{new} identically to RRT
- ▶ Instead of finding the closest node in the tree, find all nodes within a neighborhood \mathcal{N} of radius $\min\{r^*, \epsilon\}$ where

$$r^* > 2 \left(1 + \frac{1}{d}\right)^{1/d} \left(\frac{\text{Vol}(C_{free})}{\text{Vol}(\text{Unit } d\text{-ball})}\right)^{1/d} \left(\frac{\log |V|}{|V|}\right)^{(1/d)}$$

- ▶ Let $\mathbf{x}_{nearest} = \arg \min_{\mathbf{x}_{near} \in \mathcal{N}} g(\mathbf{x}_{near}) + c(\mathbf{x}_{near}, \mathbf{x}_{new})$ be the node in \mathcal{N} on the currently known shortest path from \mathbf{x}_s to \mathbf{x}_{new}

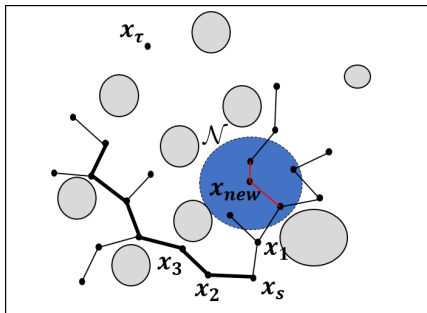
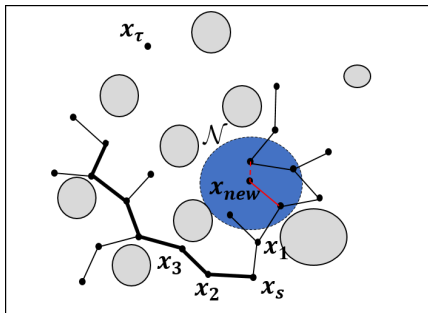
- ▶ $V \leftarrow V \cup \{\mathbf{x}_{new}\}$
- ▶ $E \leftarrow E \cup \{(\mathbf{x}_{nearest}, \mathbf{x}_{new})\}$
- ▶ Set the label of \mathbf{x}_{new} to:

$$g(\mathbf{x}_{new}) = g(\mathbf{x}_{nearest}) + c(\mathbf{x}_{nearest}, \mathbf{x}_{new})$$



RRT*: Rewire Step

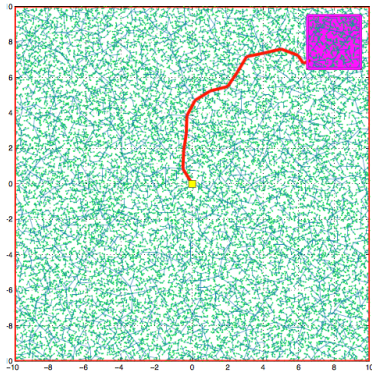
- ▶ Check all nodes $\mathbf{x}_{near} \in \mathcal{N}$ to see if re-routing through \mathbf{x}_{new} reduces the path length (**label correcting!**):
- ▶ If $g(\mathbf{x}_{new}) + c(\mathbf{x}_{new}, \mathbf{x}_{near}) < g(\mathbf{x}_{near})$, then remove the edge between \mathbf{x}_{near} and its parent and add a new edge between \mathbf{x}_{near} and \mathbf{x}_{new}



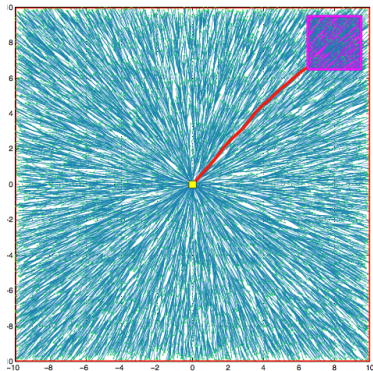
Algorithm 6 RRT*

```
1:  $V \leftarrow \{\mathbf{x}_s\}; E \leftarrow \emptyset$ 
2: for  $i = 1 \dots n$  do
3:    $\mathbf{x}_{rand} \leftarrow \text{SAMPLEFREE}()$ 
4:    $\mathbf{x}_{nearest} \leftarrow \text{NEAREST}((V, E), \mathbf{x}_{rand})$ 
5:    $\mathbf{x}_{new} \leftarrow \text{STEER}(\mathbf{x}_{nearest}, \mathbf{x}_{rand})$ 
6:   if  $\text{COLLISIONFREE}(\mathbf{x}_{nearest}, \mathbf{x}_{new})$  then
7:      $X_{near} \leftarrow \text{NEAR}((V, E), \mathbf{x}_{new}, \min\{r^*, \epsilon\})$ 
8:      $V \leftarrow V \cup \{\mathbf{x}_{new}\}$ 
9:      $c_{min} \leftarrow \text{COST}(\mathbf{x}_{nearest}) + \text{COST}(\text{Line}(\mathbf{x}_{nearest}, \mathbf{x}_{new}))$ 
10:    for  $\mathbf{x}_{near} \in X_{near}$  do ▷ Extend along a minimum-cost path
11:      if  $\text{COLLISIONFREE}(\mathbf{x}_{near}, \mathbf{x}_{new})$  then
12:        if  $\text{COST}(\mathbf{x}_{near}) + \text{COST}(\text{Line}(\mathbf{x}_{near}, \mathbf{x}_{new})) < c_{min}$  then
13:           $\mathbf{x}_{min} \leftarrow \mathbf{x}_{near}$ 
14:           $c_{min} \leftarrow \text{COST}(\mathbf{x}_{near}) + \text{COST}(\text{Line}(\mathbf{x}_{near}, \mathbf{x}_{new}))$ 
15:     $E \leftarrow E \cup \{(\mathbf{x}_{min}, \mathbf{x}_{new})\}$ 
16:    for  $\mathbf{x}_{near} \in X_{near}$  do ▷ Rewire the tree
17:      if  $\text{COLLISIONFREE}(\mathbf{x}_{new}, \mathbf{x}_{near})$  then
18:        if  $\text{COST}(\mathbf{x}_{new}) + \text{COST}(\text{Line}(\mathbf{x}_{new}, \mathbf{x}_{near})) < \text{COST}(\mathbf{x}_{near})$  then
19:           $\mathbf{x}_{parent} \leftarrow \text{PARENT}(\mathbf{x}_{near})$ 
20:           $E \leftarrow (E \setminus \{(\mathbf{x}_{parent}, \mathbf{x}_{near})\}) \cup \{(\mathbf{x}_{new}, \mathbf{x}_{near})\}$ 
21: return  $G = (V, E)$ 
```

RRT vs RRT*



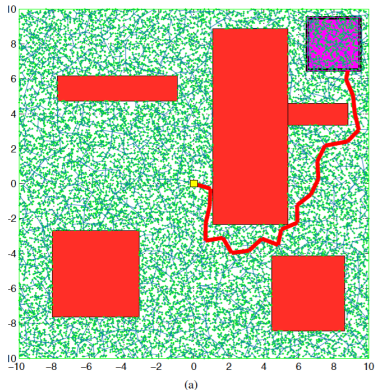
(a) RRT



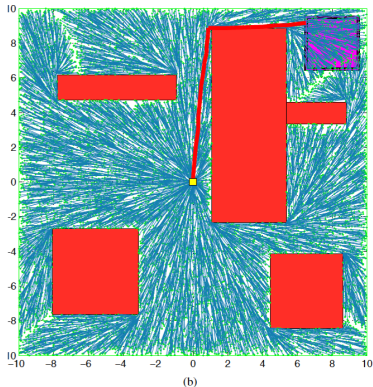
(b) RRT*

- ▶ Same nodes in the tree, only the edge connections are different. Notice how the RRT* edges are almost straight lines (optimal paths).

RRT vs RRT*



(a) RRT



(b) RRT*

- ▶ Same nodes in the tree, only the edge connections are different. Notice how the RRT* edges are almost straight lines (optimal paths).